ISSA Proceedings 1998 – Viciousness And Actual Infinity In Aristotle's Infinite Regress Arguments

Aristotle sometimes presents an infinite regress argument without showing us how its infinite regress is derived, or why its infinite regress is vicious. An infinite regress is vicious if it entails either a false statement or an unacceptable consequence. Given his omissions, we sometimes hastily grant that there is an infinite regress, and that it is somehow vicious. In this paper I will not address the derivation of his infinite regresses, but simply assume that they are entailed, and focus my attention on their viciousness.

Aristotle's notion of the infinite can appear to be involved in establishing the viciousness of an infinite regress in an infinite regress argument in the following way. An infinite regress entails the statement that (1) there are actually infinitely many entities. Given the extent to which he argues against the existence of actual infinities in his philosophical works**[i]** (especially in Book 3 of the *Physics*)**[ii]**, it is reasonable to suspect that Aristotle tacitly uses the statement, (2) actual infinities do not exist, in the infinite regress arguments where he does not explicitly discuss the viciousness. The conjunction of these two statements shows that an infinite regress entails a false statement, and consequently shows that the infinite regress is vicious.

My goal is to suggest a different interpretation: we can establish the viciousness of most infinite regresses in Aristotle's works without assuming that he tacitly uses the claim that actual infinities do not exist. The evidence that I will advance will not prove that my interpretation is the only one, but it will show that in some cases a closer fidelity to the texts obliges us to see that Aristotle's objections against infinite regresses need not follow from his notion of the infinite.

I have a number of reasons supporting this interpretation. First, in the cases where Aristotle explicitly discusses the viciousness of infinite regress, he does not make use of that claim. These are found in the *On Interpretation* 20b32-21a7,

Physics 225b34-226a6 and 242b43-53, *On Generation and Corruption* 332a26-333a15, *Metaphysics* 1006a 6-10 and1007a33-b3, *Nicomachean Ethics* 1094a18-22.
Secondly, in some cases where Aristotle doe not explicitly discuss the viciousness of an infinite regress, one can establish the viciousness without making use of his claim that actual infinities do not exist. I will describe different ways in which one can discover these alternative interpretations.

In some cases the infinite regress entails an easily identifiable implicit statement that is obviously false, and that is unrelated to Aristotle's belief that actual infinities do not exist. Consider the following. Some hold that the soul is divisible, and that we think with one part and desire with another. If, then, its nature admits of its being divided, what can it be that holds the parts together? Surely not the body; on the contrary it seems rather to be the soul that holds the body together; at any rate when the soul departs the body disintegrates and decays. If, then, there is something else which makes the soul one, this would have the best right to the name of soul, and we shall have to repeat for it the question: Is *it* one or multipartite? If it is one, why not at once admit that the soul is one? If it has parts, once more the question must be put: What holds its parts together, and so *ad infinitum* (*On the Soul* 411b5-13).
The goal of this infinite regress argument is to reject the claim that the soul is divisible. If an infinite regress were entailed, it would consist of an infinite succession of unifying parts of a soul. A necessary condition for something to "have the best right to the name of 'soul'" (411b10) is that it unify all the parts of a soul. Though each one of the infinitely many parts of the soul contributes to the unification of the soul, no single part by itself makes the soul unified. Hence, none of those part satisfies the sufficient condition. So, the regress entails the false (for Aristotle) statement that there is no soul.
A further infinite regress argument occurs later in the same book.
Since it is through sense that we are aware that we are seeing or hearing, it must be either by sight that we are aware of seeing, or by some sense other than sight. But the sense that gives us this new sensation must perceive both sight and its object, viz. color: so that either there will be two senses both percipient of the same sensible object, or the sense must be percipient of itself. Further, even if the sense which perceives sight were different from sight, we must either fall into an infinite regress, or we must somewhere assume a sense which is aware of itself. If so, we ought to do this in the first case (*On the Soul* 425b11-17).

We are presented with a disjunctive syllogism one disjunct of which is supposed to imply an infinite regress. Nothing in the context of the argument addresses the viciousness of its regress. However, the infinite regress entails the false statement that there are infinitely many senses.

In *Metaphysics* 1033a24-b4 Aristotle investigates the relation between matter and form. He uses an infinite regress argument to argue that "form also, or whatever we ought to call the shape in a sensible thing, is not produced" (1033b5-6). Whatever we make is made from something else which has form. Every form is made from a prior form. Hence, the construction of any form would entail the construction of infinitely many prior forms. But this is obviously false.

In Chapter 4 of Book 3 in *On the Heavens* Aristotle argues that the number of elements in nature must be finite. He uses an infinite regress argument in Chapter 5 in a context where he is objecting against those who believe that there exists a single element: And those whose ground of distinction [among bodies] is size will have to recognize an element prior to the element, a regress which continues infinitely, since every body is divisible and that which has the smallest parts is the element (304b6-9).

The infinite regress consists of gradually smaller "elements", and so it entails that there is no smallest element. Since that which has the smallest parts is supposed to be the element, then the implicit consequence of the infinite regress is that there is no element, and this is clearly false for Aristotle.

The identification of the false statements entailed by the infinite regresses in the preceding examples are fairly easy to see, but in some cases it does require a closer examination of the context of an infinite regress argument. For example, in *Metaphysics* 1060a27-37 Aristotle explores the nature of principles (e.g. of being, unity) used to understand the world. If they are all destructible, and if every destructible thing requires a principle in order to be understood, then the attempt to understand any principle leads to an infinite regress of principles of principles of principles, etc.. Since whatever we use to understand something must itself be understood, and we understand only by means of principles, then an understanding of any thing by means of a principle requires the use of infinitely many principles. Hence an understanding of any event would be humanly impossible. But Aristotle believes that we can explain or understand some things (Aristotle presents a similar infinite regress argument at 1000b22-28).

Consider a further challenging example. In Chapter 6 of Book Z of the *Metaphysics* Aristotle inquires "whether each thing and its essence are the same

or distinct" (1031a15-16). He is concerned with this problem because the answer might help him to determine whether universals exist apart from individual things (1039a24-b19). The reason for the interest in this problem is that if a thing and its essence are one, then the thing can be known without any recourse to Platonic Forms. He arrives at the conclusion that "each thing and its essence are one and the same but not by accident, and that to know each thing is to know its essence, and so even by exhibiting particular instances, it is clear that a thing and its essence must be one" (1031b19-21). Aristotle presents an infinite regress argument to defend this position.

The absurdity of the separation [of a thing from its essence] would appear if one were to assign a name to each of the essences; for there would be another essence besides the original one, e.g. to the essence of horse there will belong a second essence. Yet why should not some things be their essences from the start, since essence is substance? (1031b29-1032a3).

Though he does not address the viciousness of the regress, the context of the argument offers a clue. Since "to know each thing is to know its essence" (1031b20-21), and essences are treated as distinct things, then to know anything entails that one knows infinitely many distinct essences. As this is impossible to realize, knowledge of anything is impossible. But of course for Aristotle this is false. A third plausible way of establishing the viciousness of an infinite regress independently of his claim that actual infinities do not exist can be found by comparing similar infinite regress arguments. In some cases Aristotle seems to appeal to his claim that infinitely many actualities do not exist, but he also presents very similar arguments without using that claim. Of course this does not prove that he does not tacitly use it in the former cases, but it does show that there is another plausible alternative justification of the viciousness. For instance, compare the next two arguments.

Next we must observe that neither the matter nor the form comes to be – i.e. the proximate matter and form. For everything that changes is something and is changed by something and into something. That by which it is changed is the primary mover; that which is changed the matter; that into which it is changed, the form. The process, then, will go on to infinity, if not only the bronze comes to be round but also the round or the bronze comes to be; therefore there must be a stop at some point (1069b35-1070a4).

Further, the process will go on to infinity, if there is to be change of change and generation of generation. For if the later is, so too must the earlier be – e.g. if the

simple coming to be was once coming to be, that which was coming to be it was also once coming to be; therefore that which was simply coming to be it was not yet in existence, but something which was coming to be coming to be it was already in existence. And this was once coming to be, so that then it was not yet coming to be. Now since of an infinite number of terms there is not a first, the first in this series will not exist, and therefore no following term will exist (1068a33-b4).

I am definitely not saying or suggesting that these arguments are analogous in form, but that they are sufficiently similar that the reason used to support the viciousness in the latter argument could also be used to support that of the former.

Further comparisons suggest that the reason that supports the viciousness of the second example can also be used in other cases where Aristotle appears to use tacitly his claim about the impossibility of infinitely many actualities. In *Heavens*, 300a27-b1, Aristotle simply asserts that the regress that is supposed to follow from the claim, for any resting object, there is some other resting object that constrains it, is "impossible" (300b2). In the *Generation of Animals*, 715b3-15, Aristotle explores the consequences where offsprings are different in kind from their parents and are able to procreate: they would procreate a different kind of creature, who would similarly procreate another different kind of creature, and so on endlessly. The resulting regress is supposed to be vicious because "nature flies from the infinite, for the infinite is imperfect, and nature always seeks an end" (715b15). In both examples Aristotle could be implicitly arguing that the regressive process must come to an end, otherwise there would be no beginning to the either process of constraining or procreating, and this is inconsistent with their actual existence.

One can discover further ways of establishing the viciousness of Aristotle's infinite regresses without appealing to his claim that actual infinities do not exist by attending to what is suggested by his incomplete evidence advanced in support of the viciousness of an infinite regress. Consider the case in the *Posterior Analytics* 72b5-14 where Aristotle rejects the claim "that there is no way of knowing other than by demonstration" because the knowledge of anything entails a vicious regress of successive demonstrations.

The only reason he gives to show that regress is vicious is that "one cannot traverse an infinite series" (75b10). But this is by itself insufficient to establish

the viciousness. However, it suggests the other reason: we must or are obliged to go through the regress of demonstrations in order to know. The conjunction of these two reasons and the statement entailed by the regress of demonstrations that there are infinitely many demonstrations entails that we do not know anything. This consequence is false for Aristotle.

My third reason why it is not always necessary to appeal to Aristotle's claim that actual infinities do not exist is that many infinite regresses are logically superfluous. For some regresses entail false statements or unacceptable consequences even if they are neither actually or potentially infinite. Consider the following examples.

(1) Person x is a man.
(2) Person x is white.
(3) Person x is a white man.
(4) Person x is a white white man.
(5) Person x is a white white white man (*On Interpretation* 20b32-21a7).

It should be noted that this regress is superfluous beyond the derivation of the first syntactic absurdity, from (4) onwards. If the infinite regress of attributes (1007a33-b3) is vicious because "not even more than two terms can be combined" (1007b2), then any extension of the regress beyond two combinations is unnecessary in order to entail an unacceptable consequence. If an essence of an essence is unacceptable, then an infinite regress of essences (1031b29-1032a3) is superfluous beyond the essence of an essence. The regress in which everything is desired for the sake of something else (1094a18-22) need not be infinite in order to entail the unacceptable consequence that all our desires are vain and empty; it just needs to extend throughout our lives (which of course are finite). The regress of senses (425b11-17) is shown to entail a false statement at the finite extension where it entails that we have six senses.

None of regresses entailing the impossibility of knowledge, understanding, or demonstration need to be infinite (72b5-14, 1006a6-10, 1031b29-1032a4, 1033a24-b4, 1038b35-1039a4, 1060a27-37, 1068a33-b4, 1069b35-1070a4). Consider a regress of successive demonstrations that are necessary in order to know anything. It need only extend a few finite steps beyond our lives, or beyond any irremediable mental exhaustion, in order to show that knowledge is humanly unattainable. Such infinite regresses are superfluous because either false statements or unacceptable consequences follow after only a finite number of steps.

Even some causal regresses or some regresses that can be interpreted as being causal need not be infinite in order to entail a false statement or an unacceptable consequence (225b34-226a6, 242b43-53, 300a27-b1, 1033a24-b4, 1068a33-b4,1069b35-1070a4). They are typically considered vicious because they entail the nonexistence of a first term that is necessary for the existence of any current term of the regress, and this in turn entails that there is no present or current term of the regress. In order to argue my point I will first apply a standard approach to an analogous example, and then show that there are different ways of establishing the viciousness of the regress even when it is only finite. Assume an infinite regress of prior steps of a walk. According to one standard approach, the infinite regress entails the impossible task that I have walked infinitely many steps in order to reach any point on the walk. The falsity of the conclusion entails that the infinite regress is vicious. According to another standard approach, this infinite regress entails that there is no beginning, but a beginning is necessary in order to reach any point on the walk, and hence, there is no infinite regress. This contradiction entails that the regress is vicious.

However, even a finite regress of prior steps entails false statements. If the walk is extended far enough in the past, and if we assume a uniform pace, it will follow that I began walking before I was able to walk, or before I was born, or even before the universe can into existence. In each one of these cases the regress is finite and entails a false statement. Consequently, a finite regress of prior steps can be vicious.

Analogous reasoning applies to most causal regresses. Here is one way of showing this. Assuming that the universe came into existence at some finite point in the past, then prior to that point in time all physical objects at the macroscopic level did not exist. Thus, if there were a finite causal regress that extended beyond that point, it would follow that such objects existed before the universe came into being. Given the logical absurdities entailed by these finite causal regresses, they are vicious. If one is troubled by the assumption about the beginning of the universe, one could proceed in a similar way without that assumption. For example, many things as we know them today did not exist at some finite time in the past (e.g. plants, humans, insects, etc.). Any finite causal regress whose terms consist of such things can be extended far enough into a past where such things did not exist as we know them to day. For instance, humans did not exist in some remote past, but a finite causal regress of humans, entailed by a regress formula such as “Every parent has a prior parent”, can be

extended to a time when there were no humans. Since this regress entails that there were humans at such a time, the finite regress is vicious.

Given my defense of the three reasons in support of my belief that Aristotle's notion of the infinite is not necessarily involved in establishing the viciousness of his infinite regresses, why is it so tempting to appeal to that notion? I suspect that there are a number of reasons that work together.

First, Aristotle does discuss extensively his notion of infinity, and it does seem reasonable that it would be in the background of most arguments involving infinite regresses.

Secondly, some of his infinite regress arguments are not easy to analyze, and so it is much easier just to appeal to his notion of infinity in order to justify the viciousness of infinite regresses.

Thirdly, given these difficulties and the fact that not all infinite regress arguments are important, it is not clear whether it would be worth the time and effort to find alternative justifications of the viciousness.

Fourthly, the usual reading of Aristotle's works does not require a comparison of infinite regress arguments, and the arguments tend to be far apart; so it is not easy to recall the arguments in which the viciousness of their infinite regresses can be justified on a reason other than the impossibility of actual infinities.

It is in part due to this failure to compare the infinite regress arguments in his philosophical corpus that one can be disposed to overgeneralize from the few cases (e.g. 1012b19-22, 715b3-15) where the viciousness of an infinite regress can appear to be justified by the claim actual infinities are impossible. This mistake illustrates that, when seeking to theorize on a particular kind of argument, we need to compare many instances of that argument type while paying careful attention to the context of their presentation. Such a comparison can help us to see more clearly the variations that can arise, and to prevent us from squeezing all the arguments into a same mold.

In summary, I have defended three reasons in support of the conclusion that Aristotle's notion of the infinite is not necessarily involved in establishing the viciousness of infinite regresses. For in the cases where the discussions of viciousness is explicit, he does not make use of his notion; in the cases where it is implicit, I have proposed alternative ways of establishing their viciousness while retaining fidelity to the context of the infinite regress arguments and to Aristotle's philosophical corpus; and finally, I have shown that some regresses need not be

infinite in order to be vicious.

NOTES

i. At 208a5-24 he refutes arguments for an actual infinite; at 318a21 he argues that things are only potentially infinite. He gives five reasons for the existence of the infinite at 203b15-24, and discusses problems of asserting or denying the existence of the infinite at 203b30-207a31. He believes that his "account does not rob the mathematicians of their science, by disproving the actual existence of the infinite in the direction of increase, in the sense of the untraversable. In point of fact they do not need the infinite and do not use it" (207b28-30). Numbers are not actually infinite for Aristotle (1083b37-1085a2).

ii. All references and quotations are from Barnes (1985).

REFERENCES

Barnes, J. (Ed.) (1985). *The Complete Works of Aristotle.* Princeton, N.J.: Princeton University Press.

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