ISSA Proceedings 2014 – Denying The Antecedent Probabilized: A Dialectical View

Abstract: This article provides an analysis and evaluation for probabilistic version of arguments that deny the antecedent (DAp). Stressing the effects of premise retraction vs. premise subtraction in a dialectical setting, the cogency of DAp arguments is shown to depend on premises that normally remain implicit. The evaluation remains restricted to a Pascalian notion of probability, which is briefly compared to its Baconian variant. Moreover, DAp is presented as an exam-question plus evaluation that can be deployed as a learning assessment-instrument at graduate-level.

Keywords: affirming the consequent, delay tactic, denying the antecedent, dialectics, inductive logic, *modus ponens, modus tollens*, probabilistic independence, probabilistic relevance, retraction, subtraction

1. *Introduction*
We treat the evaluation of DAp, a probabilistic version of what classical logic correctly treats as the formal fallacy of *denying the antecedent* (DA), i.e., the deductively invalid attempt at inferring the conclusion *~c* from the premises *a->c* and *~a*, where a stands for antecedent, *c* for consequent, and ~ for negation. Examples include:

1. Had my client been at the crime scene (a), then he would probably be guilty (c). But he wasn't (~a), so he probably isn't (~c).
2. If the lights are on (a), then probably someone's at home (c). But the lights are out (~a), so probably no one is (~c).
3. If the product sells (a), then our marketing measures should probably be trusted (c). But it doesn't (~a), so measures should be reviewed (~c).

Here,
(1) states a counterfactual conditional ("had"),
(2) an indicative one ("are"), and that in
(3) might even sustain a deontic reading ("should"). Disregarding such differences, we proceed to treat such DAp-arguments on the following schema, its

formal version becoming clearer soon:

(DAp) If a then probably c. But not a, so probably not c.
Pf(c)=Pi(c|a)>Pi(c). But Pi(a)=0, so Pf(~c)>Pi(c).

As should be uncontroversial, if natural language instances of DAp instantiate a probabilistically valid inference, or argument, then only if the relevant probability values are right. A probabilistic version of modus ponens (MPp) can be stated as the conditional probability of c given a, i.e., P(c|a), where P(c|a) directly depends on P(~c|a) whenever *P(c|a)=1−P(~c|a)* holds, which is the complement-relation of Pascalian probability (see Sect. 5.3 on the Baconian). A probabilistic version of denying the antecedent (DAp), *P(~c|~a)*, contrasts by depending on not one, but three values: *P(c|~a), P(a), P(c).* This asymmetry between MPp and DAp is mirrored by one between probabilized versions of modus tollens (MTp) and affirming the consequent (ACp), not being treated here (see Oaksford & Chater, 2008; 2009).

As will be seen below, since particularly P(c|~a) is necessary to evaluate DAp, but need not be readily available from context, evaluations of DAp regularly remain conditional on analysts' assumptions with respect to P(c|~a). Our main objective is to present one such assumption—broadly one of relevance, referred to as AR, below—then trace AR's effects on arguers' dialectical commitments, in a context where PROPONENT (PRO) argues MPp, and OPPONENT (OPP) responds with DAp. On assumption, PRO can respond to OPP's DAp either by retracting or subtracting prior commitment; the first proves to be a delaying-tactic, and the validity of OPP's DAp is shown to depend on commitments reconstructed for PRO.

We introduce DAp as an exam-question (Sect. 2), then discuss the choice of logic (3.1), the projection of linguistic forms onto logical forms (3.2), and the retraction vs. subtraction distinction (3.3). Having provided an evaluation (4), we argue for the plausibility of AR (5.1), explain how retraction delays interaction (5.2), and briefly contrast this broadly Pascalian result with a Baconian notion of probability. Our conclusions are in Sect. 6.

2. *DAp as an exam-question*

An evaluation of a probabilistic version of denying the antecedent (DAp) in a dialectical setting might be assigned as an exam-question, such as the following, where PRO argues MPp in lines 1 and 2, to which OPP responds, in line 3, by denying PRO's antecedent, and subsequently raising the claim in line 4, thus arguing DAp. Assuming OPP to have the last word—OPP-statements "trump" PRO-statements— PRO's response options are limited to either of those in lines 5a or 5b, provided OPP is committed to PRO's claim in line 1. So, in line 6, can PRO reasonably deny OPP's claim in line 4?

(1) PRO: a makes c more probable.
(2) PRO: a is the case.
(3) OPP: a is not the case.
(4) OPP: So, not c is more probable.
(5a) PRO: I retract (2).
(5b) PRO: I subtract (2), i.e., I agree to (3).
(6) PRO: But I disagree with (4).

Task: Assume that (3) trumps (2), i.e., that OPP has the last word, and that OPP commits to (1). Evaluate line (6) as reasonable, or not, vis-à-vis (1-4), for both the variants (5a) and (5b). Trace and justify additional assumptions.

We now present a task-solution that presupposes an evaluation of DAp vis-à-vis a Pascalian notion of probability.

3. *Evaluating DAp*

3.1 *Choice of Logic*
As holds generally for argument-evaluation, an evaluation of DAp proceeds via a projection of natural language material (aka linguistic form) onto a logical form, itself provided through analyst-choice among available logics. The logic employed below is inductive, consistent with the Kolmogorow-axiomatization of probability, thus modeling a Pascalian notion of probability. As our evaluation of DAp holds relative to this logic only, external criticism of the evaluation should elaborate on inadequacies in the Pascalian notion of probability, if any (see Sect. 5.3).

3.2 *Linguistic and Logical Form*
The application of logical forms (Lo-F) to linguistic forms (Li-F) yields a reconstruction of Li-F at Lo-F level, technically a projection of the Li-F onto the Lo-F. Analysts must subsequently ask: Is a particular Lo-F validity-assessable, i.e., is the projection *complete*? It will be only if the Li-F readily provides information necessary to evaluate the Lo-F with respect to validity. Conversely, incomplete projections only require analysts to add information at Lo-F level**[i]**. Once

completed, the evaluative result may then be read-off, and transferred to the Li-F. The yield is an evaluation conditional on information added.

To appreciate the projection of statements containing ‘probable’ and its cognates, compare the Li-F and potential Lo-F instances, below, where $Pi(c|a)>Pi(c)=1-Pi(\sim c)$ states the initial probability of c given a, Pi(c|a), to exceed the initial probability of c, Pi(c), which equals one minus the probability of the logical complement, ~c, since $P(\beta)=1-P(\sim\beta)$ holds, and similarly for conditional probabilities: $P(\beta|\alpha)=1-P(\sim\beta|\alpha)$.

Above, we had seen PRO to utter the Li-F ‘a makes c more probable’ in line (1). Onto which Lo-F, now, should this utterance be projected?

(i) a makes c more probable – $Pi(c|a)>Pi(c)=1-Pi(\sim c)$
(ii) a makes c more probable than not c. – $Pi(c|a)>Pi(\sim c)=1-Pi(c)$
(iii) … than not c given a. – $Pi(c|a)>Pi(\sim c|a)=1-Pi(c|a)$
(iv) … than not c given not a. – $Pi(c|a)>Pi(\sim c|\sim a)=1-Pi(c|\sim a)$

The Lo-F in line (i) yields perhaps the most faithful projection, as its content most closely mirrors that of ‘a makes c more probable’. While (ii) to (iv) need not be implausible candidates, they nevertheless add content to PRO’s utterances. We return to (i) in Sect. 4.

Except for the point-probability $Pi(c)=Pi(\sim c)=0.5,$ the utterances in (i) and (ii) mutually and directly imply their negations. After all, (i) compares *Pi(c|a) to Pi(c), so Pi(c|a)* is also compared to Pi(~c), the latter being the complement of Pi(c), as in (ii). Similarly, (iii) compares Pi(c|a), again merely internally, to its complement, Pi(~c|a). In contrast, (iv) compares *Pi(c|a) to Pi(~c|~a),* which, importantly, does not directly dependent on Pi(c|a). Note that Pi(~c|~a) had, in Sect. 1, been seen to state a probabilistic version of denying the antecedent (DAP).

On the assumption that contents expressed by *Pi(a), Pi(c), Pi(c|a), and Pi(c|~a)* are contingent, when Pi(c|~a) cannot simply be obtained from PRO’s Li-F, then Pi(c|~a) should be stipulated in view of PRO’s commitments with respect to Pi(a), Pi(c), Pi(c|a), effectively compensating for cases where PRO avoids an explicit commitment with respect to Pi(c|~a). Sect. 4 will identify one such compensation, consisting in an assumption of relevance assumption (AR). First, we turn to PRO’s dialectical options in lines 5a and 5b (see Sect. 2).

3.3 *Retraction vs. Subtraction*

A non-formal version of the retraction vs. subtraction distinction is found, among others, in Godden & Walton (2004). In probabilistic terms, to retract amounts to PRO no longer holding a commitment with respect to the probability of a. As we now argue, retraction would only be represented unfaithfully as a PRO-update to the unspecific commitment Pf(a)=[0,1], where the subscripted 'f' indicates the final probability after retraction. To subtract, in contrast, amounts to having stated that a is false, and can be represented as a PRO-update to the specific commitment P(~a)=1.

One may assume that, having used MPP at time t0, PRO is at time t1 committed to *Pi(c|a)>Pi(c) and Pi(a)=1*. After retraction, her commitments at t2 could update to Pi(c) and Pf(a)=[0,1], where [0,1] marks the closed interval from zero to one, including the end-points, and Pi(c) is the prior probability of c. Alternatively, at t2, PRO's commitments could update merely to Pi(c). In the first case, given Pf(a)=[0,1], PRO cannot meaningfully maintain a commitment to *Pi(c|a)>Pi(c), for if Pf(a)=[0,1]* and Pi(c|a)>Pi(c) together entail anything, then they entail the probability of c given a to be greater than the probability of c, for any value of *P(a)=1−P(~a)=[0,1]*. But this is incompatible with the probability of a impacting on the probability of c. So a could not, in any standard sense, remain relevant to c, for a would now raise the probability of c come what may, given any probability-value of a, including 0 and 1 (see Sect. 5.2). To avoid as much, retraction should be modelled such that, at t2, PRO updates her commitments merely to Pi(c).

After subtraction, PRO's commitments with respect to a have been updated from Pi(c|a)>Pi(c) and Pf(a)=1, at t1, to Pi(c|a)>Pi(c) and Pf(~a)=1, at t2. They now starkly contrast with PRO's commitment at t1. Such flipping—aka 'take it back and claim the opposite'—makes it conditionally relevant for PRO to incur a comparative commitment with respect to Pi(c|~a) vs. Pi(c). Note that this is unlike the case of retraction. In both cases, of course, OPP may well ask PRO to compare Pi(c|~a) with Pi(c). In the exam-case (Sect. 2), this comparison was not made.

What may one reasonably assume about this comparison on behalf of PRO? Introduced as part of the evaluation of DAP in the next section, the assumption (AR) compares Pi(c|~a) with Pi(c). Along with other assumptions, AR will be seen to yield the very conclusion OPP seeks to establish with her DAP argument: Pf(~c)>Pf(c).

4. *Conditional evaluation of DAp*

4.1 *PROPONENT and OPPONENT commitments*

In evaluating the OPPONENT's DAp, one supposes that 'if a then c', i.e., a→c, can be interpreted probabilistically such that P(a→c)=P(c|a), an assumption referred to as 'the equation' (Oaksford & Chater, 2008; 2009). One should start from the weakest possible PROPONENT-commitment in this context (see Sect. 3.2), namely that a provides some support to c, as expressed in (7). Again, Pi(c) marks the initial or prior, and Pf(c) the final or posterior probability.

(7) *Pf(c)=Pi(c|a)>Pi(c)* – [PROPONENT-commitment]**[ii]**

As we saw, if inductive support is measured over the closed interval from 0 to 1, and reflects a Pascalian notion of probability, then a degree of support for a proposition α entails that of its complement via *P(α)=1−P(~α)*, and likewise for conditional probabilities via *P(α|β)=1−P(~α|β).* Moreover, Pi(c|a) is given by the principle of conditionalization (PC), aka the definition of conditional probability:

(PC) *Pi(c|a)=P(c&a) / P(a)* – [definition of conditional probability]

Since *P(c&a)=P(a|c)P(c)*, by substitution, the PC yields Bayes' theorem (BT)**[iii]**, to which we return in Sect. 4.3:

(BT) *P(c|a)=(P(a|c)P(c)) / P(a)* – [Bayes' theorem]

With retraction (see Sect. 3.3), the support for c in the absence of a can only depend on the prior probability Pi(c). So, if conditionalization on a results in Pi(c|a)>Pi(c), as stated in (7), then retracting a leaves the probability of c at its prior value, Pi(c). This is what Godden and Walton's (2004) claim—that retraction does not incur new commitments—amounts to when using probabilities. As OPP was to have the "last word" (see Sect. 2), one is concerned not with retraction, but with subtraction of a, i.e., conditionalization on ~a. Hence, OPP is committed to (8), which says that ~a is negatively relevant to c, as ~a makes ~c more probable than it was initially:

(8) *Pf(~c)=Pi(~c|~a)>Pi(~c)* – [OPPONENT-commitment]

Already in genuinely probabilistic contexts, where *0<P(α)=1−P(~α)<1*, the inequalities in (7) and (8) depend on suitable probability values. As the next subsection shows, such values need not be readily available in a given natural language context.

4.2 *Finding* $Pi(\sim a|\sim c)$

To illustrate the issue, assume that—unlike the extremal cases in Sect. 2, where either $P(a)=0$ or $P(a)=1$—PROP assigns $0.5<Pi(a)<1$, so that a is more probable than not, and moreover choses the likelihood, $Pi(a|c)$, such that $Pi(c|a)$ is rendered sufficiently high for the purpose at hand, i.e., beyond some threshold, t, to which we return in the next section. But assume also that PROP remains uncommitted to the exact value of $Pi(c)$. Therefore, $Pi(c)$ need not be fixed, but can in fact range over the interval satisfying $Pi(c|a)>Pi(c)$ given the chosen likelihood, $Pi(a|c)$. To reach a probabilized dialectical scenario, assume finally that PRO responds to OPP's objection by adopting OPP's claim that $0.5<Pi(\sim a)<1$. When evaluating this move, one must conditionalize on $Pi(\sim a)$ to find $Pi(\sim c|\sim a)$. Because of PRO's loose stance with respect to $Pi(c)$ before hearing OPP's objection, however, that $Pi(a)>0.5$, and that $Pi(c|a)$ were deemed sufficiently high simply does not entail a definite value for $Pi(\sim a|\sim c)$, nor only values that—upon conditionalization on ~a—leave $Pi(\sim c|\sim a)$ sufficiently low (see Sober, 2002). But some such discrete value is required to calculate with this instance of Bayes' theorem: $Pf(\sim c|\sim a)=(Pi(\sim a|\sim c)Pi(\sim c))/Pi(\sim a)$. See Oaksford and Chater (2008; 2009) and Wagner (2004) for an analytical characterization of the bounds that arise when letting $0.5<P(c|a),P(\sim c|\sim a)<1$, so that both terms count as probabilistically supported, or confirmed, if $0.5<P(a),P(\sim a)<1$.

The commitments in (7) and (8) are here treated as contingencies, and so do not express general truths about probabilistic support relations between antecedents and consequents. Hence, particularly OPP's desired conclusion—that ~c is sufficiently probable given ~a—won't follow from any old assignment of probability values, even if $0<P(\alpha)=1-P(\sim\alpha)<1$. The next subsection supplies information that leaves OPP's claim—that $Pf(\sim c|\sim a)>Pf(c|\sim a)$—acceptable through introducing the assumption AR on behalf of PRO.

4.3 *Bayes' Theorem, Jeffrey Conditionalization, and AR*

In our example in Sect. 2, $Pi(a)$ and $Pi(\sim a)$ were assigned the values zero or one. In both extremal cases, however, premise subtraction remains ill-defined in the context of Bayes' theorem. After all, when $P(a)=1$, then a is treated as indubitable, upon which the theorem ceases to offer guidance for the subtraction of a; likewise when $P(\sim a)=1$. In fact, subtraction of what is beyond doubt does widely count as an arational move in this context, a move BT does not guide one way or another. Therefore, rather than employ BT, one can turn to Jeffrey conditionalization (JC) in order to address premise subtraction (see, e.g., Jeffrey, 2004):

(JC) *Pf(c)=Pi(c|a)Pf(a)+Pi(c|~a)Pf(~a)* – [Jeffrey conditionalization]**[iv]**

In our case, when the proponent claims that a makes c more probable (see Sect. 2), she can be assumed committed to $Pf(c)>t^{3}Pi(c)$, where t is a threshold given by a probability value arbitrarily smaller than Pf(c), and at least as large as Pi(c). Further, if Pf(a)=1 and so Pf(~a)=0, i.e., a is true, then (JC) reduces to its left hand term:

(9) *Pf(c)=Pi(c|a)Pf(a)>t*

As an assumption of relevance (AR) that will be crucial for our evaluation, the proponent's initial claim—that a raises the probability of c to a value above some threshold t—may be assumed to entail the following:

(AR) If ~a (also) raises the probability of c, then at most to t, i.e., Pi(c|~a)£t.

Sect. 5.1 will argue why it is reasonable to assume AR on behalf of Pro. Let us first complete the evaluation of DAp.

4.4 *Evaluative result*

When, per our example-case, a is subtracted because a is deemed false, i.e., Pf(~a)=1, and so Pf(a)=0, then—in analogy to (9)—JC reduces to its right hand term:

(10) *Pf(c)=Pi(c|~a)Pf(~a)£t*

Because Pi(c|~a)=1−Pi(~c|~a), it follows for the standard threshold of probabilistic support t=0.5 that, upon subtracting a, i.e., Pf(~a)=1, the value of Pf(c) falls below t only if Pi(~c|~a)>t.**[v]** The evaluation, therefore, depends not only on the initial assumption Pf(c)>Pi(c), as stated in (5), but additionally on AR—i.e., Pi(c|~a)£t—and t=0.5, which together effectively state OPP's desired conclusion (i.e., line 4 in Sect. 2). After all, once Pi(c|~a) falls to, or below, the value 0.5, then c can no longer receive sufficient support in the event that ~a, since—analogously to (9)—we have it that *Pf(~c)=Pi(~c|~a)P(~a),* and so if *P(~a)=1, then Pf(~c)=Pi(~c|~a).*

Hence, rather than *Pf(c)=Pi(c|a)>Pi(c)*, as in (7), PRO would have had to be committed to:

(11) *Pf(c)=Pi(c|a)>t>Pi(c) and Pi(c|~a)£t*, for t=0.5,

for OPP to establish probabilistic support for ~c by subtracting a. Therefore, with a view to the example in Sect. 2, (5b) is unreasonable given AR. In contrast, line (5a) is at least not immediately unreasonable. But, as Sect. 5.2 argues, (5a) delays the evaluation that becomes available under AR.

5. *Discussion*

This section briefly discusses why AR is reasonable, shows retraction to be a delaying-tactic, and inquires whether the evaluative result transfers to a non-Pascalian notion of probability.

5.1 *The reasonability of AR*

Recall that, because the example in Sect. 2 lacked information on Pi(c|~a) that our inductive logic did require in order to evaluate DAp, Sect. 4.3 had introduced an assumption of relevance (AR) on behalf of PRO, namely *Pi(c|~a)£t for t=0.5.* The evaluative result (Sect. 4.4) was then seen to depend on AR. Evaluating AR requires considering whether PRO can deny AR, provided she is committed, at t1, to both *Pf(c)=Pi(c|a)>Pi(c) and Pi(a)=1*, then retracts only the latter commitment by updating, at t2, to P(~a)=1 (see Sect. 3.3). A straightforward way of addressing this consists in considering if PRO remains consistent were she to deny AR. As we saw, Pi(c|a)>Pi(c) expresses that a is positively relevant to c. So, at t1, does PRO incur a contradiction were she to commit to Pi(c|a)>Pi(c), but reject Pi(c|~a)£Pi(c)?

What if PRO were to reject Pi(c|~a)£Pi(c), i.e., accept Pi(c|~a)>Pi(c), and so be committed both to Pi(c|a)>Pi(c) and to Pi(c|~a)>Pi(c)—in words: both a and ~a raise the probability of c. In this case, were a and ~a to provide the same probabilistic support to c, i.e., Pi(c|a)=Pi(c|~a)>Pi(c), then PRO would well have avoided the commitment that c and a are probabilistically independent—which is expressed by Pi(c|a)=Pi(~c|a). But without the assumption AR qualifying the support that a and ~a lend to c as a differentially large support, the question would arise why PRO had initially offered a in support of c, when ~a could have served as well. Hence, not so much to remain consistent, but to remain relevant: at t1, if ~a shall provide some support to ~c, then such support should be lower than the support that a confers onto c, exactly as expressed by AR.

In contrast, interpreting PRO's Li-F 'a makes c more probable' from the outset to mean 'a makes c more probable than not c given a', i.e., *Pf(c|a)>t£Pf(~c|a)*, necessitates setting the threshold to t=0.5, since *Pf(c|a)=1−Pf(~c|a)*. Moreover, if P(~a)=1, then OPP's conclusion Pf(~c|~a) takes a value greater than t, which in turn shows how PRO's subtraction of a, i.e., the change in commitment from P(a)=1 to P(a)=0, establishes, or concedes, the cogency of OPP's DAp.

Besides AR, the two complement-relations P(α)=1−P(~α) and P(β|α)=1−P(~β|α) for conditional probabilities remain crucial to our evaluation, because information not provided at Li-F was inferred by means of these relations. We discuss both in Sect. 5.3, and now proceed to argue that, here, retraction is at best a delaying-tactic.

5.2 *Retraction as a delaying-tactic*

In Sect. 3.3, we had seen that retraction amounts to avoiding a commitment with respect to the probability of a, including a lose commitment such as P(a)=[0,1]. Assume, then, that PRO has successfully avoided as much, and so is committed, at t2, merely to Pi(c|a)>Pi(c), and Pi(c). As argued above, this set of commitments allows PRO to disagree, in line (6) of Sect. 2, with OPP's claim that Pf(~c)>Pf(c). The disagreement is not immediately unreasonable because, after retraction, information necessary for OPP—and for analysts—to establish Pf(~c)>Pf(c) was seen to be unforthcoming from PRO's commitments.

As PRO had, at t1, claimed that P(a)=1, even after retraction, OPP can demand that PRO commit to some comparison of Pi(c|a) with Pi(c|~a) vis-à-vis the threshold t=0.5, provided this OPP-move is not otherwise blocked. Moreover, provided that PRO would act in an irrelevant manner were she to reply with a comparison other than AR—as argued in Sect. 5.1—then OPP can still establish her claim in line (6). So when interlocutors can elicit commitments and criticize irrelevant claims, retraction merely delays the OPPONENT's conclusion, minimally by one turn.

These considerations all highlight the role of the assumption AR. As AR compares Pi(c|~a) and Pi(~c|~a), being terms directly related via the complement principle Pi(c|~a)=1−Pi(~c|~a), it should be of interest to compare this evaluation with a Baconian notion of probability, where this principle does not hold.

5.3 *Baconian probability*

Jonathan L. Cohen (1980) has coined the term 'Baconian' for a notion of probability whose central assumptions differ from those of its Pascalian counterpart. Crucially, Baconian probabilities are non-additive; therefore, the above complement-relations do not generally hold, and also conditional probabilities may be defined differently. Being ordinal values, Baconian probabilities can be compared but, unlike Pascalian probabilities, one cannot readily add, subtract, multiply, or divide them (see Cohen, 1980; Schum, 1991; Hajek & Hall, 2002; Hájek, 2012; Spohn 2012).

For our case, which was seen to depend on AR, it may thus well be the case that, for instance, $Pi(c|a)=0.8>Pi(c)=0.5$, while nevertheless $\mathrm{Pi}(\sim c|a)=0$, rather than $\mathrm{Pi}(\sim c|a)=0.2$, as the complement-principle of the Pascalian calculus has it. So, a may make c more probable to an extent e, without it being entailed that the probability of ~c given a is calculated as $1-e$. The scale of Pascalian probability runs upward from disproof to proof, while the Baconian scale runs upward from non-proof, or no evidence, to proof (see Cohen, 1980). Evidence for α having been provided thus remains compatible with no evidence having been provided for its negation, ~α.

Baconian probability is particularly applicable to the legal domain. For instance, the probability that a defendant is guilty may be assumed to be determined by evidence typically provided by the prosecution. Is the prosecutor's evidence less than conclusive, however, then whatever evidence is lacking will, on the Pascalian notion, entail a corresponding disproof of the defendant's guilt (compared the first example in Sect. 1). On the Baconian notion, in contrast, the prosecutor's evidence in support of the defendant's guilt compares independently to evidence forwarded on behalf of the defendant's innocence, or lack thereof. In the absence of such evidence, then, the probability of the defendant's innocence would (hopefully) register at 0. And if disproving evidence is forwarded, the probability of the defendant's innocence (hopefully) registers at values independent of the probability of the defendant's guilt.

We cannot claim to have done any justice to the Baconian notion of probability, but may nevertheless conclude that the evaluative result (Sect. 4) need not without further ado transfer to a non-Pascalian notion of probability. So analysts are required to decide, for the particular case and in view of the natural language material, whether a Baconian or a Pascalian notion of probability is more appropriate.

6. *Conclusion*

Presupposing a Pascalian notion of probability, we have provided an analysis and evaluation for probabilistic version of arguments that deny the antecedent (DAp). Stressing the effects of premise retraction vs. premise subtraction in a dialectical setting, the cogency of DAp arguments was shown to depend on a premise that normally remains implicit, namely $Pi(c|\sim a)$£t, for $t=0.5$, which we had identified as a relevance assumption. Moreover, premise retraction was shown to be a delaying-tactic as long as the opponent can ask the proponent to incur new commitments. Generally, the cogency of DAp arguments was seen to depend on commitments ascribed to the proponent. As we have stressed, the evaluative result is restricted to a Pascalian notion of probability, which was briefly compared to its Baconian variant. On these qualifications, the abstract version of DAp presented in Sect. 2 can be deployed as a learning assessment-instrument at graduate-level.

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NOTES

i. Other tweaks are subtracting information, and changing its order (permutation); both modifications, however, normally presuppose possessing information that is necessary for an evaluation.

ii. (7) leaves open the exact degree of support; one of its measures, $S(c|a)$, can be defined as: $S(c|a)=Pi(c|a)-Pi(c)>0$ (Korb, 2003, 44; Howson & Urbach, 1993, 117).

iii. Dropping the subscripts, BT comes in two equivalent versions:

(BT) $P(c|a)=(P(a|c)P(c)) / P(a)$

(BT*) $P(c|a)=P(a|c)P(c) / (P(a|c)P(c)+P(a|\sim c)P(\sim c))$

One reaches BT* by substitution in BT, since P(a)=P(a|c)P(c)+P(a|~c)P(~c). Here, P(a|c) and P(a|~c) express likelihoods, namely the probability of a given c, and the probability of a given ~c, respectively. P(a|c) can be read as the impact of a on P(c). P(a|~c) is also known as the false positive rate. To express the classically valid modus pones inference with (BT), if aÉc is true, then P(c|a)=1. So the rate of exceptions, P(~c|a), is zero since P(c|a)=1−P(~c|a). See Oaksford and Chater (2008; 2009).

iv. (JC) has the posterior probability of the conclusion, Pf(c), depend on the posterior probability of the antecedent, Pf(a)=1−Pf(~a), as well as the prior probabilities Pi(c|a) and Pi(c|~a), the latter two terms being mutually independent. Jeffrey conditionalization generalizes the Bayesian theorem, where (BT) corresponds to the limiting case that arises by setting one of JC’s summands to 1. To verify, recall that Pf(c)=Pi(c|a). Since P(a&c)=P(c&a)=P(a|c)P(c)=P(c|a)P(a), by substitution, if Pf(a)=1, then the expression Pf(c)=Pi(c|a)Pf(a)+Pi(c|~a)Pf(~a) reduces to Pf(c)=Pf(a&c), so Pf(c|a)=P(a|c)P(c)/P(a) becomes Pf(c|a)=Pf(a&c). The case is analogous when Pf(~a)=1.

v. To assume that Pi(~c|~a)>t for t=0.5 amounts to a probabilized version of the conditional perfection strategy—where, as part of the analysis, -> is perfected to <->—for this very assumption renders the conditional ‘a then c’ convertible, probabilistically speaking.

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