ISSA Proceedings 1998 – Conditional Reasoning

*1. Introduction: logic and argumentation***[i]**
I would like to start with a pronouncement: I believe that logic is and must be a essential tool for the testing, classification and explication of arguments as well as reasonings. Specially, it's the job of logic to distinguish between *valid* and *unvalid* arguments, as well as between *good* and *bad* reasonings. In this sense, the main role of logic in the theory of argumentation is not descriptive nor explanatory, but normative. I think this deontic dimension is necessary for drawing the boundaries between rhetoric and argumentation, which are the boundaries between proving and persuasion.
This solemn beginning is not just to release myself. From my point of view, it's not a passing fancy to remind the normative character of logic. A logical entity may be used as a model for a physical or mental entity, but in any case it's a ideal model. In the case of argumentations, this means that it has not the properties of the real entity, but the properties that we think the real entity *ought to have.*
The aim of this lecture is to provide a definition as well as a brief explanation of a special kind of reasonings which I will call "conditional reasoning". This definition must be understood as the first step to a general theory of conditional reasoning which is not explained here, and whose main bricks are the logical theory of conditionals (see Vilanova 1995, Vilanova 1996). The term "conditional reasoning" is a new one in the literature, so some people will look to it in surprise. Nevertheless, a lot of authors have defined similar notions, and all of them have showed a big interest in the topic. Later on we will see some examples. For the moment it's enough to note that the medieval logicians use a very similar notion when defining the "dubium proponitur" (I propose to doubt) arguments: arguments where something evident or firmly believed is negated, in order to know what theoretical consequences it would produce.

2. A "prima facie" definition
I will begin by explaining the two words included in the title. I would distinguish two senses of the word "reasoning":
i. *Cognitive or Psychological sense:* a mental event consisting in a thinking

process directed to the resolution of some problem. This is the customary sense of the word reasoning, the sense we mean when we talk about the reasonings that our neighbours make, or the reasonings that our politicians don't make. In other words, this is the action to which we compel when we say "use your brain, reason!"
ii. *Logical sense*: a triad D,C where P is the set of premises, C is the conclusion, and D is a deduction of C from P. P, D and C are set of sentences. They may belong to a formal language (for example, the language of first order logic with some supplementary symbols as identity, modal symbols, conditional operators…). But they may belong to a natural language. Sherlock Holmes stories, as well as scientific books, are full of reasonings in this sense where the sentences belong to a natural language. The main difference between an argument and a reasoning is that in a argument the premises are supposed to be true. On the contrary, in a reasoning the premises don't need to be true; they are just those propositions not proved in the deduction.

We may understand a reasoning in the logical sense as a *model* of a reasoning in the cognitive sense. In other words, we use linguistic entities (propositions) for modelling mental entities. Some philosophers and psychologists, as Fodor, think that mental entities are also linguistic entities belonging to a special language, the language of mind. If they are right, then we ought to speak about public linguistic reasonings (second sense) as models of private linguistic reasonings (first sense).

Regarding the second word in the tittle, there are two important notions related to the word "conditional":
i. *A conditional statement* (in English) is a statement of the form "If …, then… " or a statement that can be paraphrased in this form. For example,
(1) If I were a rich man, I would buy a lorry is a conditional statement. But also
(2) When the sun rises, the cock sings
(3) You eat, you pay.

ii. A conditional operation (*or operator*) is a function from pairs of statements to statements. For example, the material implication "_" is a conditional operator which gives, for every pair of statements, B a statement "A_B" such that "A_B" is true if an only if A is false or B is true.

We use conditional operators for modelling conditional statements. In other words, we define conditional operators that represent what the words “if-then” express in English. A conditional operator _ would be a good model of a class of English conditional statements _ (it’s very probable that there is more than of one significant class of statements) if the truth value of A_B depends on the truth-value of A and B in the same way in which the truth value of “If A, then B” depends on the truth value of A and B for all the statements of _.

We can give now a *prima facie* definition of a conditional reasoning. A conditional reasoning is a two-steps reasoning such that:
– The first step is the formulation of a hypothesis (a supposition, a not-known-to-be-true proposition).
– The second step is the deduction of consequences from the hypothesis.

An example will help to understand this definition. Suppose that I want to go to the cinema this night, but I have not car. Now it’s half past nine, and the night session starts at ten o’clock, so I will not arrive to the cinema on time going by foot. My brother suggests to use my bicycle. Immediately, I put my brain to work. First of all, I make the supposition of my using the bicycle, so I imagine myself taking the bicycle out of the garage, driving it… Then, I try to infer that I’ll arrive to the cinema on time for the movie. I calculate how much time I would need to arrive to the cinema, and I discover that it would take at least twenty minutes to go from my home to the cinema. Then I remember that I have to inflate the tyres, and I calculate that I will need at least fifteen minutes to pump them up. I conclude that I will not arrive on time and I decide to see the television show at home. This example shows two important features of conditional reasonings. First, it shows that conditional reasonings are guided towards a specific goal (the deduction of a statement). Second, it shows that sometimes they miss their goal, their fail to prove the desired statement.
Conditional reasonings very often come into sight in everyday life. Some times we are not sure about the truth-value of a proposition, or we just want to talk about the future, or we want to talk about the way things could happen. In all these cases we have a proposition which is not true (perhaps it’s not false also), so we start our reasoning by stating a hypothesis. Really, conditional reasonings are essential in common sense reasoning. In the tradition of the logical positivism and the analytic philosophy the paradigm of reasonings (in the logical sense) were reasonings taken from formal languages.
Philosophers in this tradition use these kinds of reasonings in the logical sense for

modelling the inferences typical in scientific research, but the greater part of the reasonings we make in everyday life resisted to analysis. Today many investigators show a special interest in modelling common sense reasonings as the bicycle one, which require more powerful and expressive logics.

3. Some references

As I said, up to a point, conditional reasoning is a novel notion. This means that at least I don't know of the existence of any precise definitions of this concept. But some authors, specially in the field of conditional logic, have defined related notions. I feel that it's noteworthy to give some examples of these related notions, in order to see that the "novelty" is not "too" new.

Donald Nute (Nute 1980: 5-16) use the notion of "hypothetical deliberation". For Nute this is the kind of inference we follow when we have to manage to extract conclusions from a false statement A. According to Nute, the hypothetical deliberation has the form of a mental experiment. We design alternative situations where the statement is true, and that are *reasonable* enough. If we want to known if another statement, B, follows from A, we try to design a reasonable alternative to the actual situation that makes A true, and where B is false. If we arrive to such a definition, B follows from A. If we fail to arrive to such a situation after a good piece of deliberation, of we judge that it's not possible to elaborate such a counterexample, then we conclude that B follows form A. The basic point in Nute's theory is the word "reasonable". As Nute explains, our standards about what is reasonable change depending on the occasion. There are situations that are reasonable in a context but not in a different context. Even in a concrete context, the reasonability criteria are not precise: they don't use to be explicit, and only vaguely they are presupposed in their totality. In any case, there are two boundaries for the alternative situations: those preposterous, crazy situations, and those "ad hoc" situations that confirm very clearly B.

Pollock use the term "subjunctive reasoning" to name the common feature of a set of phenomena that traditionally has been deemed philosophically problematic (Pollock 1976: 1-4). These phenomena include counterfactual statements (Conditional statements whose antecedent is false), but also laws of nature, causal statements, dispositions and probability statements. The "subjunctive" element of these phenomena is the recurring to state of things, events of situations that doesn't happen in the actual world, and consequently we have to resort to verbs in the subjunctive mood to express them. Lets take a disposition as

example:
(4) This piece of gold is soluble in acid. In order to explain the meaning of this sentence we make use of a subjunctive sentence:
(5) If this piece of gold were submerged in water, it would be dissolved.

Following Pollock, subjunctive reasoning presuppose a “strange metaphysically suspicious” kind of logically contingent necessity:
“To say that the Watergate scandal would not have occurred had Kennedy been president in 1972, seems to be to assert some kind of necessary connection between those two states of affairs. If there were no such connection, how could the occurrence of the one possibly effect the occurrence of the other? This same kind of necessity rears its ugly head repeatedly through subjunctive reasoning. The necessity in question is clearly not logical necessity, but what other kind is there?” (Pollock 1976: 2)

Explaining this “strange kind of necessity” is, according to Pollock, the key to the understanding of subjunctive reasoning. I think that the word “subjunctive” in Pollock’s notion plays the same role that the word “conditional” in my notion of conditional reasoning. Likewise, the word “hypothetical” in Nute’s account, “conditional” in Stalnaker’s notion of “conditional deliberation”, and “counterfactual” in Lewis’s formal model, all of them point to the same kind of phenomena. A phenomena which is closely related to conditional sentences.

4. Conditional sentences
Conditional sentences play an important role in conditional reasonings. On the one hand, rational agents, while following a conditional reasoning, make implicit or explicit use of conditional statements: “if it were the case that…, then it would be the case that…”. On the other hand, when expressing conditional reasonings, human resort to conditional statements. In the bicycle example, in order to communicate to my brother my inference, I will say something like that:
(6) If I want to use the bicycle I’ll have to inflate the tyres; But if I inflate the tyres it’ll will take me ten minutes, and if I go by bicycle from here to the cinema, it will take another twenty minutes….

Furthermore, conditional reasonings produce conditional statements. In the bicycle example, my conditional reasoning ends when it reaches the conditional statement:
(7) If I use the bicycle I’ll not arrive on time.

The result of the reasoning, its effect, is a conditional statements. Conditional statements are processes directed to the production of conditional statements, but there are other ways to produce conditional statements. For example, and restricting the discourse to material implication, if we have a disjunction:
(8) I'll go to the cinema or I'll stay at home we can use the rule of disjunctive syllogism:
(9) If I don't go to the cinema, I'll stay at home.

Sometimes we use the Aristotelian syllogism, when we have as premises two conditional statements such that the antecedent of one them is the consequent of the other one. For example, from:
(10) If the bell sings, the calf lows.
(11) If the calf low, the cow moos.

I can infer:
(12) If the bell sings, the cow moos.

Which is the way conditional reasonings make conditional statements? A conditional reasoning follows the pattern of the implication introduction rule. In the application of this rule, we start by making some assumption A. Then we deduce another sentence B from A, the premises and the set of all tautologies. When we arrive to B, we cancellate A (it can not be used in later deductions) and we conclude that A implies B. If we represent the making of a assumption with a horizontal line, and the cancellation of the assumption which another horizontal line connected to the previous one by a vertical line, an application of the rule of the implication rule goes as follows:
A
…
…
…
B
A_B

Which kind of operator is ->? This is, still, an open question. For sure it is not material implication, at least in common sense reasonings. A -> B amounts to the truth of B or the falseness of A. This is a very weak relation between A and B. Quoting Pollock, there may not be any kind of "necessary connection". Even it's possible that A and B express two isolated, completely unrelated events, for example “Galilee was Italian” and “Venus is a planet”. But when we arrive to a conditional statement by using a conditional reasoning we conclude something stronger, we conclude that A entails, carries on or causes B. Material implication won’t do!

The operators defined in modal conditional logic are meant to express these kind of conditional relations between sentences. The counterfactual implication of the V-logics defined by David Lewis(->), or the conditional implication defined by Stalnaker (>), are good candidates for at least some classes of conditional reasonings. Let’s be precise about this point. If the conditional sentences produced in conditional reasonings (If A, then B) have the syntactical properties and the truth value conditions of one of these operators (A B, or A -> B), then this operator may be selected for modelling conditional reasoning. Actually, I think that one single operator is not enough for all the relevant conditional sentences. In Vilanova (1995), (1996) and (1998) I propose a set of four conditional operators, and I pretend that they are enough to give an account of a great proportion of the natural language conditionals produced in everyday reasonings. This is not the place to describe these operators or to discuss their respective merits. I just want to point out that if we take some of these operators as the formal counterpart of the “if-then” English words, we need to allow in the deduction of B from A some inferences that traditional logic doesn’t include. We need to allow the use, for example, of some rules that fall back on semantics, as the presentation of interpretations as counterxamples, as well as the use of iconic representations, w-arguments, inductive inferences… Modelling these strategies of reasoning is not easy, and a lot of work has still to be done. The notion of hyperproof of Etchemendy and Barwise progresses in the line of including these strategies, and some important and recent logical developments, as nonmonotonic, fuzzy logic or epistemic logic, invite also to optimism. In any case, we will omit this problem in this paper, and we limit ourselves to classical logic.

5. *Formal definition*

A *Conditional reasoning* is a sequence <P,S,O,G,D,C> such that:

A *Conditional reasoning* is a sequence $\langle P,S,O,G,D,C\rangle$ such that:

1) $P \cup S \cup O \cup D \cup C$ is a set of sentences.

2) P (the premise set) is the union of the set B (background knowledge) and F (the frame premises).

3) S is a sentence called the *supposition*, and O is a set of sentences called *secondary suppositions*. G is a set of sentences called the *goals*.

4) D (the deliberation or deduction) is a sequence of sentences $\langle A_0, A_1, \dots, A_n\rangle$ such that:

4.1.) $A_0 = S$

4.2.) for any A_i ($1 \leq i \leq n$), rather $A_i \in O$ or A_i is the result of the application of some deduction rule to some sentences belonging to $\{A_m / m<i\} \cup B$

4.3.) For every $A \in O$, there is a cancellation of A in D.

5) C is $S \rightarrow A_n$.

We say that a conditional reasoning is SUCCESSFUL if $A^N \in G$.

The set B may be not explicit or extensionally defined. For example, it may be just “background knowledge” or “a description of the actual world”. The set F include the explicit premises, what we take as the point of departure of the problem we want to solve. It may include the description of the situation involved in the problem (its frame) or the particular context of argumentation. In any case, F (and in extreme cases also B) may be empty. The set O include the assumptions we make in the course of deduction that are cancellated before ending it.

These secondary suppositions mark conditional reasonings that take place in the course of the main reasoning. An example: in the bicycle reasoning I may consider two alternative routes, one through the park and another by the main road. Then I calculate how much time each of them will take. So I open a secondary assumption (I’ ll go through the park), I calculate the time and conclude twenty minutes.
I open another secondary assumption (I will go by the main road) and conclude another twenty minutes. So I conclude that it will take twenty minutes.
I think that the definition is clear by itself, and I will not extend myself in explanations. It’s more interesting to look for some interesting cases of conditional reasonings.

6. Applications
In this paragraph, I’ll suggest the application of the former definition to some typical human reasonings.
Evaluation of counterfactual statements.
Counterfactual statements are statements whose antecedent is false. One typical problem we have to resolve is determining if some concrete counterfactual statement is true or not. When dealing with counterfactual statements we can not

contrast the conditional relation with the real world, because the event expressed by the antecedent doesn't happen in the reality, so we have to make a "mental experiment". Ramsey proposed a test for the truth of a counterfactual :

– First, revise your beliefs in order to make the antecedent true.

– Then, if the consequent is true according to your revised beliefs.

If the consequent is true, the counterfactual is true. The counterfactual is false otherwise.

A application of the Ramsey test is a sort of conditional reasoning. In this case, B is a description of the actual world (complete in the ideal case), F is empty, S is the counterfactual antecedent and G is the set composed by the consequent and the negation of the consequent. In the ideal case, the reasoning is always successful: the counterfactual is true when An is the consequent; the counterfactual is false when An is the negation of the consequent. In real situations we must take into account a third option: those cases where the rational agent is not able to deduce the consequent nor its negation, and accordingly he still doesn't know the truth value of the counterfactual (the conditional reasoning is not successful).

Prediction Problems.

Prediction problems may be seen as the search of an answer to the next question: "Lets suppose that such event happens, what will it follows?". In this case G is the set of all sentences, because we look for any consequence of the event. G is, of course, the event. F is the description of the present state of affairs, from which we try to deduce the forthcoming events. B is the rational agent's background knowledge. This background knowledge include what the agent knows about the "physics" of the world, as well as what we may call "common sense" knowledge, general information of a more doxastic than scientifical character.

Decision-making problems.

Decision-making tasks are inquiries about the consequences of our actions. We may see them in terms of this question: If I decide to do this action, will I get some of my objectives?. In this case, S is the action I'm thinking on do, G is the set of the subject's goals or ends. We suppose S and we make deductions till we arrive to one of the goals. F and B are as before.

Diagnosis (Ginsberg 1986).

Diagnosis may be explained in terms of the conditional relation between the cause (disease) and the observation (symptom). The question here is: Would this disease produce this symptom? F is the description of the system, S is the possible cause and G is the observed failure.

Hypothetical-deductive method.

In scientific research it's usual to try out a theory by inferring from it propositions which are verifiable by observation or experimentation. In this case, S is the thesis and we try to infer from the thesis a verified statement B or its negation ¬B, so G is the set composed by B and ¬B. If An is G then the theory is explanatory. If An is ¬G, then the theory is unvalid.

NOTES

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