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1. Introduction
There is a lot of definitions of argumentation systems used for different purposes. In some of the papers one can distinguish simultaneous use of the notion of argument in two senses: as a proposition that is an argument for the thesis and as a proof method. For the second we use argumentation functions and argumentation strategies to characterize it. In that model many of the nonclassical logics are definable in natural way.
The argumentation processes may be considered as games over a judge opinion (Gargov, G. & Radev, S. 1987a), (Vreeswijk, G. 1993). The winning (price) of such a game is the judge verdict. The judge may be of different forms and different structure:
1. In the case of discussion the judge is the common knowledge (and opinion) of both players,
2. in the football play type (or the administration) the judge consists of a set of judges and possibly is structured hierarchically (higher instance),
3. In the chess the judge is the rules of the game except the case when the champion is elected.

Moreover the judge knowledge (opinions, evaluations, believes) may differ from the knowledge of the players and possibly his believe system may change during the discussion (game). From the other hand the players may be honest or not and their honestness may be included in the calculation of the price (judge opinion) or not. Also the judge may be honest or not. These possibilities determine a couple of different games and deductive problems. Few of the interesting (and simple) examples are investigated in the present paper. For more complicated cases we need more time/place then the limit of that paper.

2. Arguments and argumentations
In logical investigations we often treat semantics generated in the process of justifying some statements by means of other statements. The latter are usually called arguments for the former. At the beginning (Gargov, G. & Radev, S. 1986) we tried a simplified approach based on the following assumptions:

1. the argumentation is one-step, i.e. the arguments are already with a definite truth value determined by their meaning,
2. the scheme of evaluation is consistent in the sense that no already evaluated statement should be given argument.

Such simplified considerations lead us to the so-called argumentation functions generating truth definitions given some basic semantics (Vakarelov, D. 1972), (Gargov, G. & Radev, S. 1986, 1987a, 1987b), (Radev, S. 1996). The corresponding logical systems (treated in the cited papers only at the propositional level) turned out to be small (3 or 4-valued) many-valued logics. Later we tried to extend the treatment to more dynamic situations when the arguments for a given statement are also questioned and this gives rise to an iterated procedure of argument evaluation.

3. Argumentation systems

The argumentation systems are powerful instruments for decision making. In the present paper we investigate more complex argumentation systems instead of simple ones because of the complexity of the problems we have to decide in the implementations. Nevertheless we start with the construct of the simplest argumentation systems.

Elementary argumentation system is a triple EA=Q,A, where:
1. P,Q are finite sets of nodes (propositions); elements of P and Q are propositions; elements of P are the arguments and of Q – the thesis,
2. A is a finite set of binary (argumentation) relations in P×Q such that A,P,Q are disjoint sets. For every a -> A and every p -> P and q -> Q if q -> a, then we say that p is an argument for q in a-sense (the expert a believes that p is an argument for q), or briefly: p is a-argument for q.

Evaluated elementary argumentation system is 5-tuple E=Q,A,V,s where:
– Q,A is an elementary argumentation system,
– V is a set of values,
– s:P -> V is a partial mapping called semantical (justification) function.

Without loose of generality we may suppose that either the set P consists of these arguments we know their values, or the mapping s is total. It is easy to see that every partial mapping may be extended to total. When P -> Q= -> (in the elementary argumentation systems no argument may be a thesis) it is natural to suppose that only arguments from P are evaluated. In the case when not P -> Q=

-> (some arguments may have their arguments too, hence for some A it appears as an argument and as a thesis) the situation is more complicated.
(1) Typical example of elementary argumentation systems is the connection between propositional and predicate logics. Let P be the propositional language for arithmetic and Q be the set of quantified (closed) formulas. The meaning of every formula of the form -> xA(x) (-> xA(x), respectively) depends on the meaning of the corresponding formulas A(0),->,A(n). Hence for every i the formula A(i) is an argument for the formula -> xA(x). Naturally the set of all arguments for -> xA(x) is infinite, but one can find a finite set of arguments that characterize the thesis -> xA(x) (respectively the example that proves -> xA(x) (Gargov, G. & Radev, S. 1987b)).
In many cases as: generalized quantifiers (Dale, R., Hovy, E. Rosner, D. & Stock, O. (Eds.) 1992), implication formulas (Gabbay, D. 1976), induction, etc. one can find examples of evaluated argumentation systems or equivalent notions. The semantical function s is defined only on the basic formulas and inductively extended on the complex ones. The extension depends on the law (strategy) we choose for it. In the previous example the law is: “Accept the thesis -> xA(x) only if you accept all its arguments”. Analogously the modus ponens law says: “Accept B if you accept A and A -> B.” The greatest part of the logical semantics are based on such laws. Hence the corresponding argumentation systems characterize these logics (Gargov, G. 1987).

4. Semantical functions

For the semantical functions we suppose that are total, because of the trivial extension of every partial function with a new value “?”. Also it is natural to base the investigations on the classical semantical functions, c i.e. such that V={0,1} and

– c(A -> B)=min(c(A),c(B)),
– c(A -> B)=max(c(A),c(B)),
– c(¬ A)=1-c(A).

Sometimes the same effect is obtained by the down-closed classical semantical functions (Gargov, G. 1987) for which we suppose only:
c(A -> B)=1 implies c(A)=1 and c(B)=1,
c(A -> B)=1 implies c(A)=1 or, c(B)=1,
c(¬ A)=1 iff c(A)=0.

The down-closed semantical function is based on the language-generator: only the generated formulas are evaluated, all other are free of logical values. Here under the language-generation mechanism we understand the model in which the language is not apriory given but is generated in parallel with the proof. In that model it is not the case that the language, proof methods and semantics are given independently and we prove theorems about their relations. More precisely is to say that we prove properties of a given logic-language-information complex.

5. Strategies

There are different strategies to obtain the “logical” value of the thesis from these of it’s arguments. It is natural to think that the basic arguments have only two possible values – 0 and 1 (false and true). The atomistic principle says that if there are more logical values then there have to be arguments and strategies that allow us to reach these values. As we show some of the many-valued logics are based on the 2-valued and the corresponding strategies (Gargov, G. 1987), (Gargov, G. & Radev, S. 1986), (Radev, S. 1996).

The strategies that one can find in the literature are in different forms. Using any strategy one can define a corresponding semantical function for the elements of the argumentation system. The more complex strategies may be build from the simplest. Hence first we consider the simplest ones based on the two element set of values V={0,1}. Between simplest strategies for 1 – ‘’True’’, 0 – ‘’False’’, ! – ‘’Defined’’ and ? – ‘’Not defined’’ one can observe in the human decision processes there are:

1 -> 1 – the thesis is accepted if all it’s arguments are true (Lukasiewicz, J. 1920),

1 -> 1 – the thesis is accepted if at least one of it’s arguments is true (Blikle, A. 1991),

1¬ -> 0 – the thesis is accepted if it lacks false arguments (Thomason, R. & Horty, J. 1988),

1¬ -> 0 – the thesis is accepted if not all it’s arguments are false,

0 -> 0 – the thesis is rejected if all it’s arguments are false (Blikle, A. 1991),

0 -> 0 – the thesis is rejected if at least one of it’s arguments is false,

0¬ -> 1 – the thesis is rejected if not all arguments are true,

0¬ -> 1 – the thesis is rejected if it lacks true arguments as well as the less logically ones:

11 – the thesis is accepted if the greatest part of it’s arguments are true (Polya, G. 1954)

00 – the thesis is rejected if the greatest part of it’s arguments are false (Polya, G.

1954)
1 -> ! – the thesis is accepted if all its arguments are defined (Blikle, A. 1991),
0 -> ? – the thesis is rejected if some of it's arguments are not defined (Dewey, J. 1910)
0 -> 11 – the thesis is accepted if there is no more then one of its arguments false (Verheij, B. 1995)

Some of these strategies seems the same. Naturally in the 2-valued case some pairs of strategies produce the identical results. We mention all these strategies because they are prepared for many-valued cases in which "there is no true argument" and "all arguments are false" means different things. In the many-valued semantics these strategies will not only "accept" or "reject". In the brackets one can find a paper (not the unique) where such a strategy is used (not always consciously). It is surprising how often the authors omit the information about the deduction strategies they use. Usually under strategy we shall mean any combined strategy, for instance the combination of the strategies "accept if all are acceptable" and "reject if all are rejectable" (1 -> 1 and 0 -> 0) give us a maximal strategy – to be true or false that strategy needs all the arguments to be of the same value (maximal assurance). Intuitively such a strategy is used when the decision maker have to have "maximal assurance" of his decision. Analogously the combination "accept if all, reject if exists" (1 -> 1 and 0 -> 0) is a conjunctive strategy because of the conjunctive representation in logic; (1 -> 1 and 0 -> 0) disjunctive strategy. When the basic logic is not the classical one we consider more complicated strategies; for instance the maximal strategy in the three valued case consist of: "accept -> if all are ->, e.g. the strategies (1 -> 1 and 0 -> 0 and ? -> ?) where the new logical value "?" corresponds to "not determined". In general the strategy says how to compute the logical value of the thesis from the logical values of its arguments.
Note that the additional value? in the maximal strategy allow us in the next argumentation step to obtain 1 using some of the nonlogical strategies, while after the conjunction strategy that will be impossible.
Interesting form of argumentation system is (Wittgenstein, L. 1961) where tree-like argumentation system is mixed with classical logic proofs and even the hypertext grammars.

6. Argumentation systems

Now we are ready to introduce the notion of argumentation systems. It is time to take into account the difference between arguments from P and conclusions from Q. For that we suppose that a family L of different languages is given and that a foundation principle holds: "No argumentation process may have arguments from different levels." In the argumentation systems we can make proofs as in the logical systems. Again some proofs may be totally correct, but now sometimes the arguments for the thesis may be not sufficient for the acceptance of the thesis. Let us build successfully the argumentation systems from the elementary ones to the complex.

7. New semantics

There are many possibilities for the organization of the argumentation. Respectively the justification (argumentation) functions may have different properties. For instance for every A the set j(A) may be: finite, empty, m-element, less then m-element.

Also we suppose that the justification function is logical – for every A and B we have:

j(A -> B)={C -> D: C -> j(A) and D -> j(B)}

j(A -> B)={C -> D: C -> j(A) and D -> j(B)}

j(¬ A)={ ¬ C: C -> j(A)}

The logical justifications will be called argumentations.

From all these objects we define a new semantical function t(s,j, – >):Q -> V -> where V -> (possibly V -> V) is a new set of truth values. In other words the value of the formula A is obtained by the ->-type calculation of the s-values of it's arguments from j(A). For instance, if j(A)={B,C} and s(B)=0 and s(C)=1 and _ is the conjunctive strategy, then t(A)=0 because there is one 0, while by the maximal strategy it is ? because there are 0 and 1, whence the value is neither 1, nor 0 (Gargov, G. & Radev, S. 1986). Obviously we have the following trivial but important fact.

Fact: If the justification function j is 1-element, then the produced semantics t is equivalent to the given semantics s independently of the strategy.

Even the contradiction argumentation strategy (1 -> 1 and 0 ->0) produces a semantics in the case when j is 1-element. That's why in the classical logic it is sufficient to show that the axiom is true and to have true all its consequences.

Note, that in most investigations some of these objects are mixed: sometimes j and s are mixed in one element, almost always L=L -> and V=V ->, very often s and t are the same function. The last is natural, because in almost all logics s=t. In the logics, however, we investigate consequence relations, in algebra we investigate functions and operations, in linguistics we investigate grammars and languages, etc. And simultaneously we fix all other objects to eliminate “the noise”.

8. Formal definition

An argumentation system consists of a family of elementary argumentation systems, hence the arguments came from different languages and values and semantical functions are also families. Argumentation system is a 5-tuple AS=L,A,V,N in which:

– S is a set of strategies,
– L is a (set of) work language(s),
– A is a set of argumentations,
– V is a set of values,
– N is a set of semantical functions

Elementary argumentation is a pair *O where I is a set of arguments (expressions from some of the languages from L) and O is an argument (expression from another language from L). The argument O is the conclusion and the arguments I are the premises of the elementary argumentation.*

Evaluated elementary argumentation is any elementary argumentation *O extended with a semantical function s_N such that s(i) is defined for all i* -> I.

Argumentation node is an evaluated elementary argumentation extended with a strategy -> S.

New semantical function t is defined for the argumentation conclusion in the argumentation node after the calculation of the values s(I) of the premises with the strategy s.

Argumentation process is a tree of argumentation nodes, the root of which is a thesis, and every predecessor of a node is an argument for that node. In other words the nodes are labeled by arguments from A and by strategies from S. The premises of a node are all the arguments for that node. A successor is accepted if the argumentation strategy in that node gives an acceptable value from the values of all the predecessors (arguments) for that node. The thesis is accepted if the argumentation calculation accepts the root. Note that in some cases the acceptance may be connected with more then one designated values from V.

Intuitively the argumentation system is a mixture of evaluated elementary argumentation systems. There are languages instead of sets of arguments and consequences, there are sets of truth values instead of one complex truth values. The truth values are practically decision possibilities. In the complex decision the situation is changed and thus in the new situation the decision maker has new set of possibilities.

9. Argumentation logics

In the argumentation systems, the connections between facts is not so strong as in the logical systems. Hence we may collect all the facts, that are (possibly) in contradiction in one family. These facts later are organized in small consistent theories and the consequences of these theories are the arguments for the decision. The logical way is to use many-sorted languages and the connections between sorts are axiomatically introduced in the system. In the complex argumentation systems we have as many sorts as formulas (propositions) using every proposition as an justification identifier. Hence the classical logic approach is not immediately applicable in the argumentation systems. From the other side we have nonclassical logics in which one can manipulate inconsistency information in a logical way. In these logics the logical values may be considered as possible decisions. Naturally there are connections between argumentation systems and the many-valued nonclassical logics (Gargov, G. 1987).

Many of the multi-valued logics correspond to some argumentation systems. All 3 and 4-valued logics are based on the mentioned strategies, hence are simple. It seems that the relevant logic has the most complicated strategy.

10. Some simple argumentation games

First we suppose the simplest case when there are only two dramatic personae of the game: P, or Proponent, and O -Opponent. The game is played on a language L where all the relevant assertions are made. In turn P and O choose statements and put them forward to the other. If there arises an uncertainty the other player asks a question and depending on the answer continues or stops the game. The players support their statements by arguments (we may assume that the arguments are given by some argumentation function). These arguments though are to be evaluated by the other player. Put very briefly the game may have the following outcomes:

1. one of the players wins unconditionally – when he has found a true (in the opponent’s sense) argument making all opponents arguments false;

– one of the players wins “relative to some ambiguity”
– when he has found a true argument while his opponent’s arguments are either false (but not all of them) or undefined;
2. a disqualification of one of the players
– when he has produced false arguments while the other has failed to produce anything true but has not given obvious falsities;
3. a mutual disqualification
– when both produce false arguments;
4. a true tie game (real contradiction in the game)
– when both have true arguments;
5. an undefined game
– when the arguments of both players are undefined (not evaluated by the opponent);
Thus we have a kind of 9-valued logic governing the truth evaluation of a statement.

11. Judges

The Judge is the most important person in every game. Interesting is the fact that in most investigations (Bvivedi, M.N. 1886), (Davidson, D. 1990), (Hintikka, J. 1976, 1984), (Gabbay, D. 1976), (Ricoeur, P. 1976) there is no judge in the games. In some of the papers the game is not investigated or even there is no word about the argumentation game considered. The judges seems are the authors and they propose their judge strategies. We want judge to be “honest”, but we think the honest means “We are write!”.**[i]** Hence we try to make him “honest” in our sense and respectively propose the arguments for that. Because the opponent is not “honest” we prepare the arguments to persuade the “True”. The symmetry says the opponent thinks in the same manner. Hence the Judge have to make a truth from these two different truth. His strategy is based on questions (if he is allowed to ask, because the reader of newspapers, the listener of the politicians, the TV observer, etc. have no possibilities to ask questions.).
In some discussion games (Vreeswijk, G. 1993) the judge is a part of the rules of the game. For instance when the repetition is forbidden and the initial semantics of the judge is in the given argumentation system then his semantics is based on a some form of the empty argumentation rule – if the opponent has no new argument then the proponent wins.
Following almost the same arguments as Hintikka (Hintikka, J. 1984) we introduce in the discussion two players P and O. Whenever there is a trivial

strategy for both players – “My thesis is the only right.” – we introduce the third player – the Judge (J). The Judge is the most important dramatic person in every discussion. Every discussion is made only for him and the winning of the play is his opinion. The propositions of P and O are J-evaluated. In the classical theory of games (von Neumann, J. & Morgenstern, O. 1953) the judge is introduced as the price. In the logical games the judge is introduced as the rules of the game (logics). Hence the game has full information about his evaluations. In the economic behavior the money the player win are good form of a judge if there is no inflation. But in many discussions about the future the judge cannot be defined because only one of the possibilities is realized. Let us suppose that the elementary game is a pair of propositions of the players. The semantical function of the judge J may have three possibilities – 1 (true), 0 (false), and ? (unknown). The Judge have to evaluate 9 possibilities. From the sports we have natural property of the judge – he have to show the winner. The values of the judge’s verdict are of the following types:

1. wins unconditionally (knock down)
2. wins by points
3. tie game

for both players where there are three tie games (0,0), (1,1) and (?,?). If the judge has the “disqualification” possibility then every 0 means disqualification, the value (0,0) means disqualification of the both players. If he has possibility to give two equal medals, then (1,1) means that. The only “pat” situation is the value (?,?). In some games that possibility is eliminated by dividing the judge onto odd number of judges. Sometimes the “pat” has also the meaning “continue the game” like in some football elimination’s.

We may consider also the judge function as “greater then” with the natural order 1?0. Hence the judge possibilities are 3 and his decision is the usual implication table for the three-valued classical logic.

0 – ? – 1
0 – ? – 0 – 0
? – 1 – ? – 0
1 – 1 – 1 – ?

Here the judge role is to compare the argumentations of both players. If the result is 1 then P is the winner, if 0 – O and if the evaluation is ? – the game is continued. As it is easy to see the judge realizes the conjunctive strategy. A generalization of that fact is the following theorem:

Theorem. For every game there is an argumentation system with three argumentations such that the conjunctive strategy gives the value -> iff in the corresponding game the result is ->.

There is possibility to play multi-step game instead of one-step. When the game is continued the players propose the next arguments. Usually are investigated games with no repetition of the arguments. That restriction is realizable only in the simplest languages in which we can recognize when two propositions are equivalent.

The possible answers of the players are new arguments Pk and Ok such that Pk is an argument for some Pi, i k and Ok is an argument for some Oj, jr. The Judge questions in such a game are of the form “Why?” It is easy to see that the first positive (negative) answer evaluate all the arguments. The only case when new question is possible is when the Judge strategy is of the type 0 -> 11 (not accepted when the number of positive arguments is small).

It is possible the judge to be not honest even when both players are honest and the judge is their common knowledge. That is in the case when the common knowledge is defined inconsistently. To verify the judge we need a superjudge hence return to the start point – argumentation game for the judges.

12. Play without opponent

The play without opponent is simple – the judge asks for arguments and extend his argumentation system. The thesis is accepted (1) when the judge reach only true arguments. Otherwise the judge continue the questions until he construct an “acceptable” closed world. If there is a contradiction then the judge’s verdict is 0. Here we have a form of the disjunctive strategy applied. The only difference is in the second Judge – his verdict is 0 when the proponent has no arguments for the second thesis. If the judge agree with the unknown arguments and agree with the argumentation relations then the result is equivalent to the application of the disjunctive strategy.

13. Conclusions and related topics

The argumentation games with judges in fact are 3-agent systems. Multi-agent argumentation systems are powerful instrument for prediction and analysis (Biedrzycki, J., Gryczan, A. & Radev, S. 1997), negotiations (Sierra, C., Jennings, N., Noriega, P. & Parsons, S. 1998). and other fields of Artificial Intelligence.

Multi-agent argumentation systems allow us to deduce from dynamic inconsistent information that is impossible in logical systems. From the other hand a lot of logical systems may be considered as argumentation systems. Usually these logics differ only on the argumentation strategies and/or justification functions. The argumentation systems allow us to compare such logics in a natural way – without any changes in the formal language or axiomatic systems.

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NOTES
i. This includes not only "That what I say is true", but also "My way of thinking is true" and even "This is the only true way of thinking".

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