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1. Introduction

Some arguments have premisses which make their conclusions probable. Or so, at least, it seems. But the attempt to understand how and under what circumstances they do so has proved surprisingly difficult. Carnap's project of an inductive logic (Carnap 1962/1950) foundered on the inability to single out a unique measure function which would assign initial probabilities to each set of structurally isomorphic state descriptions (Carnap & Jeffrey 1971, Jeffrey 1980). On a Bayesian personalist approach, which goes back to F. P. Ramsey's 1926 paper "Truth and probability" (1990/1926), an initial purely subjective (hence "personal") assignment of probabilities is modified according to Bayes' theorem in the light of subsequent evidence (hence "Bayesian"); Bayesian personalism has recently had vigorous defenders (e.g. Howson & Urbach 1989, Kaplan 1996), but a critical examination by John Earman (1992) concluded that it still faces, among other difficulties, the so-called "problem of old evidence" (explaining how old evidence can make a hypothesis more probable, as the already known perturbation in the orbit of the planet Mercury evidently did for Einstein's general theory of relativity). John L. Pollock has attempted to ground a comprehensive theory of inductive reasoning and inductive argument on what he calls "nomic probability" (Pollock 1990: 25), the kind of objective probability involved in statistical laws of nature. Various authors have developed criteria for "argumentation schemata" covering such types of argument as enumerative induction (particular and general; cf. Russell (1948)), eliminative induction (inference to the best explanation), and so-called "direct inference"; such ad hoc approaches, exemplified by Grennan (1997), often seem plausible, but need justification.

In this paper, I wish to make a start on developing criteria for determining whether the premisses of an argument make its conclusion probable; we could say that such a situation is one in which the premisses "probabilify" the conclusion, so the subject of this paper is probabilification.

I propose to start from Stephen Thomas' discussion of an example in the 1997 (fourth) edition of his *Practical Reasoning in Natural Language* (Thomas 1997:

130-131). In his discussion, Thomas maintains a position adopted in print 13 years earlier (Thomas 1984: 32), even though a subsequently published paper (Nolt 1985: 56) rejected that position. It will turn out that, in this dispute, Nolt is correct and Thomas is mistaken. The textbook discussion makes clear, in a way that the earlier paper did not, why Thomas made his mistake. His reason is a seductive one, and exposing its inadequacy has, I shall maintain, some negative lessons for the evaluation of enumerative induction and, more generally, of inferences from confirmatory evidence to the probable truth of any hypothesis under investigation.

2. Thomas' discussion

Thomas discusses the strength of support given to the conclusion of the following invented argument by its premisses:

1. The fifty marbles in this bag were thoroughly stirred and mixed before sampling.

The first forty-eight marbles examined, each chosen at random, were all clear glass.

Therefore, the remaining two marbles are both clear glass (Thomas 1997: 130).

In its surface form, this argument projects a property of all examined members of a class to two unexamined members of that class. But, since the two marbles in the bag are the only unexamined members of the class, the argument by implication projects the property of being clear glass to all members of the class. Thus, although in its external form it is an example of what is commonly called "particular enumerative induction", its logic is that of what is usually called "universal enumerative induction" (Russell 1948).

Thomas writes the following about his example: "The large proportion of marbles examined and the fact that the marbles were thoroughly stirred before sampling and were chosen at random, all contribute to the strength of this reasoning. Yet the reasons do not make the truth of the conclusion totally certain. It remains possible that one (or even both) of the two marbles still in the bag is not made of clear glass. Although we can imagine that the reasons are true while the conclusion is false, this situation is unlikely. Consequently, the step from these reasons to the conclusion is rated as *strong* [emphasis in original – DH]. Unlikely as it may be, the logical possibility that a remaining marble is not clear glass (despite the fact that the first 48 drawn at random were clear glass) makes this step of reasoning less than 100 per cent certain, the highest possible degree of strength." (Thomas, 1997: 131)

In calling the step from the reasons to the conclusion strong, Thomas is claiming that the reasons if true make the conclusion highly probable, though not certain. As he puts it: "A practical measure of a *strong* [emphasis in original – DH] degree of support is that the reasons be related to the conclusion in such a way that the truth of the reasons, if they were true, would establish the truth of the conclusion with a degree of certainty strong enough to count on it with confidence for all realistic purposes." (Thomas 1997: 130)

In the earlier exchange in *Informal Logic*, Thomas maintained that the probability of the conclusion in a similar example was "well in excess of 80%," but did not explain how he arrived at such a quantitative estimate (1984:32) (In the earlier example, the premiss was that 49 of 50 marbles in an urn had been examined and found to be blue; the conclusion was that the 50th marble will also be blue.) Nolt replied that "in fact there is no way to calculate such a probability from the information Thomas gives… We can, without violating any mathematical law, assign that proposition [that the next marble to be selected from the urn is blue–DH] any probability we like." (1985: 56) Perhaps because Nolt gave no argument for his counter-claim, Thomas obviously remained unconvinced.

3. The mistake

Thomas appeals to its being unlikely "that the reasons are true while the conclusion is false" (1997:131). In other words, he regards as unlikely the following situation: 48 marbles selected at random without replacement from this jar containing 50 marbles are clear glass, while one or both of the non-selected marbles is not clear glass. Now we might estimate the value of this probability as follows. If the bag contains 49 clear marbles, this probability is 2/50, or .04.**[i]** If the bag contains 48 clear marbles, this probability is 1/1225, or .00082.**[ii]** If we are given only that the bag contains either 48 or 49 clear marbles, then the probability is somewhere between these two values, depending on what relative likelihood we assign to the two possibilities.

Since the probability of a set of mutually exclusive and jointly exhaustive events sums to 1, then by subtraction the probability of the remaining two marbles being clear glass is .96 in the one case and .99918 in the other, or some value in between if we do not know which of the two cases obtains. We might then be inclined to take one of these values, or the range between them, as the likelihood that the conclusion of our argument is true, given that the reasons are true. But this would be a mistake. For the argument corresponding to the probability of .96

(or .99918, or the range between) is the following argument:
2. The fifty marbles in this bag, of which 49 (or 48, or either 48 or 49) were clear glass and the remaining one was not (or two were not), were thoroughly stirred and mixed before sampling. The first forty-eight marbles, each chosen at random, were examined. Therefore, the remaining two marbles are both clear glass.

Although verbally very similar, that is logically a very different argument from (1), the argument whose inference we are evaluating, since it includes in the premiss the assumption that one of the original 50 marbles is not made of clear glass (or two are not) and it does not include the information that the 48 examined marbles are made of clear glass; indeed, this original premiss is incompatible with the combination of the new argument's premiss and its conclusion. What has gone wrong? In construing as we did the likelihood that the reasons are true while the conclusion is false, we have treated the problem as one of estimating the probability of a given outcome of a not yet completed stochastic (indeterministic) process. But our problem does not involve any such stochastic processes.
We are supposing that the 48 marbles have already been selected, and that they are all clear glass. There is no indeterminacy about their colour, or about the colour of the remaining marbles in the bag. So the probability we are interested in is not a probability in any frequency sense, but an epistemic probability: the degree of confidence in the truth of the conclusion which the truth of the premisses would give to a rational person.
Further, we have confused a conditional probability with the probability of a conjunction. Specifically, we have confused the probability that the conclusion is false, *given that* the premiss is true, with the probability that the conclusion is false *while* the premiss is true, where this "while" is construed as a conjunction. Given the standard constraints imposed on a probability function by the Kolmogorov axioms**[iii]**, we can equate the probability that an argument's conclusion is true, given that its premisses are true, with 1 minus the probability that the conclusion is false, given that its premisses are true. ($p(C|P) = 1 – p(\text{not } C|P)$.)**[iv]** But in general we cannot equate the probability that the conclusion is false, given that its premisses are true, with the probability that the conclusion is false while its premisses are true. (In general, $p(\text{not } C|P)$ _ $p(\text{not } C \,\&\, P)$).
The possibility of such a confusion shows that there is a danger of misapplying Thomas' general test for assessing the degree of support given to a conclusion by its premisses. Since this general test has been incorporated into other textbooks (e.g. Pinto & Blair 1993; Pinto, Blair & Parr 1993), it is worth noting the need for

care in how it is stated. Thomas proposes (1997: 135-136) the following procedure for estimating degree of support.

First, ask whether, supposing the reasons are true, there is any way in which the conclusion nevertheless could be false.

Second, if there are such ways, estimate how likely it is that the most likely of these ways is true; if there are no such ways, the argument is deductively valid.

Third, assess the degree of support of the conclusion by the reasons as the complement of this estimate: strong if the most likely counterexampling way is highly unlikely, moderate if it is unlikely but not highly unlikely, weak if it is only somewhat unlikely, nil if it is at least as likely as not. Care needs to be taken in applying the first of these three steps. One is not looking for a way in which the reasons are true *and* the conclusion false. One is looking for a way in which, *given* that the reasons are true, the conclusion is nevertheless false. The difference is subtle but, as the above discussion indicates, important.

4. Some calculations of epistemic probability

We must try, then, to calculate an epistemic conditional probability. It is important to notice that the estimation of such a probability depends not only on the information stated in the premisses, but on other background information at our disposal. For example, we assume that the number of marbles in a bag changes only by removing or adding marbles to the bag; contrast the number of drops of water in a glass, or the number of rabbits in a cage, or the number of mothballs in an open jar. We assume that no marbles left or entered the bag after it had 50 marbles in it, except for the 48 which were examined; in particular, we assume, although the argument does not explicitly say so, that none of the 48 marbles were put back in the bag. We also assume that marbles do not spontaneously change colour on their own; contrast chameleons or mood rings. We cannot ignore this information, since no calculation is possible without making some such assumptions. Such background information plays an even bigger role in more complex real cases. The need to take it into account in estimating the degree of probabilistic support of a hypothesis by evidence is Carnap's so-called "total evidence requirement" (Carnap 1962/1950: 211-213; cf. Pollock 1990: 133).

We can get some idea of the range of probabilistic support in our example by considering some assumptions which might form part of our additional background information in a real-life situation. I shall consider in turn three cases.

Case 1: Uniformity: The marbles were put in the bag from a single uniform-coloured batch from a production line. (Cf. Hume's assumption that nature is uniform. In fact, nature is not uniform in all respects, but we often have, or think we have, reason to think that we are dealing with a sortal property of a natural kind, e.g. the solubility of a pure substance in a pure substance.) In this case, the conditional epistemic probability that the conclusion is true is 1, since the premisses together with the background information entail the conclusion. Note that in this case the background information makes it unnecessary to examine more than one marble from the bag, just as in many scientific experiments or systematic observations we do not need to accumulate a large number of instances, just enough to make us confident that our lab technique was good and our measurements were accurate.

Case 2: Independence: The marbles in the bag were selected by a random process from a very large stock of marbles of various colours, the proportion of clear glass marbles in this stock being some known ratio r. (For example, the stock had 10 different colours of marble; each colour was assigned a distinct one-digit number, and a random number table was used to select by number the sequence of colours of the marbles put in the bag.) The stock was large enough that taking 50 clear glass marbles from it would not affect substantially the proportion of clear glass marbles in the remainder of the stock. The observed result is of course highly improbable in this case. But improbable events do happen. And drawing 48 clear glass marbles in succession from the bag does not change the probability, relative to the specified background information, that the remaining two marbles are also clear glass; to think otherwise is to subscribe to a fallacious form of the law of large numbers. So the probability is r that the 49th marble is clear glass, and is also r that the 50th marble is clear glass. Since, given the background information, these two events are independent, the probability of their conjunction is the product of the two probabilities, or r2.

Case 3: The travelling gambler: The marbles were put in the bag by a roving gambler who has read Thomas' textbook. This nefarious individual goes to college and university campuses where Thomas' textbook is used, and proposes a sinister betting arrangement to unsuspecting students who have read Thomas' discussion of example (1). He shows them his bag with 50 marbles, which they can count for themselves (without looking inside the bag). He invites them to draw 48 marbles from the bag. If all 48 marbles are clear glass, he offers them, if they are willing to bet a sizeable sum of money, attractive odds that the remaining two marbles are also clear glass, say 3:1 in his favour. (The students in fact believe the chances are overwhelming that the remaining two marbles are clear glass, so they are happy to give him such favourable odds.) There is always a non-clear marble in the bag. Of course, 24 out of every 25 times the odd marble is drawn among the 48, and there is no bet. But every 25th time the roving gambler cleans up. In this case, the probability of the conclusion being true, given the truth of the premisses, is 0, since the premisses and the background information together entail the falsehood of the conclusion.

These three cases collectively vindicate Nolt's claim that "we can, without violating any mathematical law, assign that proposition [in his example, the proposition that the remaining ball is blue–DH] any probability we like." (1985: 56) The additional background information supplied in each case does not alter an antecedently determined epistemic probability. Rather, it supplies enough information to enable a definite probability to be calculated at all. Since the resulting probabilities range from 0 to 1, it seems obvious that, by suitable adjustment of background assumptions, we can indeed assign any probability we like to the proposition that the remaining two marbles in the bag are clear glass, given the information in the premisses of (1). Without more information than that supplied in the premisses, we cannot attach even a qualitative degree of confidence to the conclusion, relative to the premisses.

5. The Bayesian approach

The same result obtains in the three cases if we adopt as the basis of the epistemic probability we are estimating some form of Bayesian personalism, which takes our degree of confidence in a proposition to be a function of the odds we would think it fair to give in a bet that the proposition is true, where the system of such confidence assignments is constrained at least by the probability calculus.**[v]** If we are absolutely confident in a proposition's truth, then we would think it fair to give somebody who doubted its truth as high odds as the person wished, e.g. a million to one. If we think it just as likely as not that the proposition is true, then we would think it fair to give odds of 1:1 that the proposition is true. In general, if the odds we think fair to give on the truth of a proposition are x:y, then we have a confidence of x ÷ (x + y) in the truth of the proposition, and vice versa.**[vi]**

The Bayesian approach allows us to use Bayes' theorem to calculate the epistemic probability that a hypothesis H is true given certain new evidence E, a probability generally referred to as the *posterior probability*, provided we are given three other epistemic probabilities, each construed as an assignment of a degree of confidence to a proposition. First, we need the *prior probability* of the hypothesis, that is, the probability that the hypothesis is true, given our background information independently of the new evidence. (I shall call this "p(H/K)", where p is the probability function, H is the hypothesis and K is our background information apart from the new evidence.) Second, we need the *posterior likelihood*, the likelihood of the evidence on the assumption that the hypothesis is true, again assuming the same background information which we have independently of the new evidence. (I shall call this "p(E/H & K)", where E is the new evidence.) Third, we need the prior likelihood of the evidence, the likelihood that the evidence is true on the assumption of our background information, without assuming the truth of the hypothesis under investigation. (I shall call this "p(E/K)".) Bayes' theorem tells us that, if the prior likelihood is not zero, the posterior probability of a hypothesis on new evidence is its prior probability multiplied by the ratio of the posterior likelihood of the evidence to its prior likelihood:

(3) $p(H/E \& K) = p(H/K) \times p(E/H \& K) \div p(E/K)$.

The proof of the theorem rests on the definition of a conditional probability p(A/B) as the result of dividing the probability that both A and B obtain by the probability that B obtains, provided that this latter probability is not zero. If one replaces the conditional probabilities in Bayes' theorem according to this definition, one sees that the theorem is correct, provided that neither the prior probability of the hypothesis nor the prior likelihood of the evidence is zero.

In argument (1), as noted in section 2, the hypothesis can be regarded as the hypothesis that all the marbles are clear glass. Since the evidence of the first 48 marbles drawn being clear glass is a logical consequence of the hypothesis that all 50 marbles are clear glass (given implicit background assumptions such as those mentioned at the beginning of section 4 above), the posterior likelihood of the evidence is 1. Hence, in this case the posterior probability of the hypothesis will simply be the prior probability of the hypothesis divided by the prior likelihood of the evidence.

Case 1: On the uniformity assumption, the prior probability that all the marbles are clear glass is, we may suppose, some value *r*. The prior likelihood that the first 48 marbles drawn from the bag will be clear glass, given the uniformity assumption but not assuming the truth of the hypothesis, is also *r*. So the posterior probability is 1.

Case 2: On the independence assumption, the prior likelihood that any marble in the bag will be clear glass is *r*. Since our assumption makes the colour of each marble drawn independent of the colour of any other marble drawn, the prior probability of the hypothesis that all 50 marbles are clear glass is *r50*, and the prior likelihood that the first 48 marbles drawn from the bag will be clear glass is *r48*. Hence the posterior probability of the hypothesis is *r50 – 48*, or *r2*.
Case 3: On the travelling gambler assumption, the prior probability that all 50 marbles in the bag are clear glass is 0. Hence, whatever the prior likelihood may be, assuming it is not 0, the posterior probability of the hypothesis is 0.
The fact that Bayesian calculations produce the same results in all three cases as the more informal reasoning in section 4 both vindicates the Bayesian approach and increases our confidence in the informal methods of the previous section; there is, I believe, no vicious circularity in using the coincidence of results from two distinct approaches as evidence boosting our confidence in both of them, provided that neither approach is a logical consequence of the other.

6. An invalid argument schema
A tempting approach to our example is to note that the evidence reported in the premisses rules out all but three hypotheses about the distribution of colours among the 50 marbles in the jar:
1. All 50 marbles are clear glass.
2. 49 marbles are clear glass, and one is not clear glass.
3. 48 marbles are clear glass, and two are not clear glass.
One can then note that the evidence is much more likely to occur on the first hypothesis than on the second and third.
On (1), the evidence is bound to be obtained.
On (2), the likelihood of the evidence is .04, as calculated in note 1 above.
On (3), the likelihood of the evidence is .00082, as calculated in note 2 above.

Since the result we observed was inevitable on the first hypothesis but highly unlikely on each of the only two alternative hypotheses consistent with our evidence, does this result not make it highly probable that the first hypothesis is true, and thus that the last two marbles in the jar are clear glass?
The considerations advanced in the above possible solutions show that this method of reasoning is invalid, that is, that the premises do not necessarily confer a high probability on the conclusion of the argument. A Bayesian explanation of why it is invalid is that it does not take into account the prior probability of the

three hypotheses. If our background information gives (1) a very much lower prior probability than (2) or (3), the fact that the evidence is exactly what we would expect on the basis of (1), but highly unlikely given (2) or (3), is not enough to make (1) highly probable.**[vii]**

Thus, the following argument schema, though plausible, must be rejected as invalid:

4. The observed results rule out all but n mutually exclusive hypotheses.

On one of these hypotheses, the observed results were bound to occur. On any of the others, the observed results were highly improbable. Therefore, probably the first hypothesis is correct.

7. Conclusion

The probability which the premisses of an argument confer on its conclusion is the complement of the probability that the conclusion is false, *given that* the premises are true. But it is a mistake to identify this conditional probability with the probability of a conjunction, the probability that the premises are true *while* the conclusion is false. Such identification leads to serious errors in estimating the degree of support of an argument's conclusion by its premisses. Apparent commission of this mistake in contemporary textbooks shows that one must apply with care the procedure of estimating degree of support as the complement of the likelihood that the conclusion is false, given that the premisses are true.

In an enumerative induction, whether universal or particular, the conditional probability that a property observed in all examined members of a class or kind belongs to all, or to one or more hitherto unexamined, individual members of the class or kind must be assigned in the light of an evaluator's background information.

Uniformity of the examined instances of a kind with respect to some variable can make it certain, highly probable, improbable or even impossible that all the instances of the kind, or the next examined instance(s), will be similar in that respect, depending on an evaluator's background information. In some cases the background information does not permit assignment of a definite epistemic probability, or even a rough range of such probabilities, to the conclusion. In general, a hypothesis is not necessarily made highly probable by an observed result which is highly likely on that hypothesis but very unlikely on each of its competitors; the prior probabilities of each of the hypotheses under consideration must also be taken into account. **[viii]**

NOTES

i. In a sequence of 50 random selections without replacement from a bag containing 49 clear marbles and one non-clear marble, there is an equal chance of the drawing of the non-clear marble occurring at any position in the sequence, namely one out of 50; therefore, this is the probability that the drawing of the non-clear marble will occur last in the sequence, and also the probability that the drawing of the non-clear marble will occur second last. Since these events are mutually exclusive, the probability that one or the other of them will occur is the sum of the probability that each will occur, namely 2/50, or .04. This is the same as the probability that, in a random selection without replacement, the first 48 marbles drawn will be clear glass. The same result follows if, using the general multiplication rule for calculating the probability of a conjunction of events, we multiply the probability of choosing a clear marble on the first draw (49/50), the probability of choosing a clear marble on the second draw given that a clear marble has been chosen on the first draw (48/49), and so on, up to the probability of drawing a clear marble on the 48th draw if 47 clear marbles have been drawn on the first 47 draws (2/3). Here and in other calculations in this paper, I use Kolmogorov's (1956/1933) axioms of the classical probability calculus. See note 3.

ii. In a sequence of 50 random selections without replacement from a bag containing 48 clear marbles and two non-clear marbles, which we may call "marble A" and "marble B", there is an equal chance of the drawing of marble A occurring at any position in the sequence, namely one out of 50; therefore, this is the probability that the drawing of the marble A will occur last in the sequence. The probability that the drawing of the marble B will occur second last in the sequence, given that the drawing of marble A occurs last, is one out of 49. By the general multiplication rule, the probability of the conjunction of these events is 1/50 x 1/49, or 1/2450. By similar reasoning, the probability that the drawing of marble A will occur second last in the sequence and the drawing of marble B last is also 1/2450. Since the two conjoint events are mutually exclusive, the probability that one or other of them will occur is the sum of these two probabilities, i.e. 2/2450, or 1/1225, or .00082. This is the same as the probability that, in a random selection without replacement, the first 48 marbles drawn will be clear glass. The same result follows if, using the general multiplication rule for calculating the probability of a conjunction of events, we multiply the probability of choosing a clear marble on the first draw (48/50), the probability of choosing a clear marble on the second draw given that a clear marble has been chosen on the first draw (47/49), and so on, up to the probability of drawing a clear marble

on the 48th draw if 47 clear marbles have been drawn on the first 47 draws (1/3).
iii. Kolmogorov (1956/1933: 2) proposed in effect the following axioms for the probability calculus (where p is the probability function and P and Q arbitrary arguments, here construed as propositions, to which this function is applied):
(1) p(P) -> 0.
(2) If P is a tautology, then p(P) = 1.
(3) If P and Q are mutually exclusive, then p(P -> Q) = p(P) + p(Q).
Any function which satisfies these or an equivalent set of axioms is a probability function in Kolmogorov's sense.
iv. This result depends on the assumption that p(P) Õ 0. Given this assumption, we have that:
1 = p(P)/p(P)
= p[(C or not C) & P]/p(P) [conjoining a tautology]
= p[(C & P) or (not C & P)]/p(P) [distribution of disjunction over conjunction]
= [p(C & P) + p(not C & P)]/p(P) [Kolmogorov axiom]
= p(C & P)/p(P) + p(not C & P)/p(P) [arithmetic]
= p (C|P) + p (not C|P) [definition of conditional probability].
Thus, subtracting from each side, 1 – p(not C|P) = p(C|P). This proof constitutes a justification of Thomas' (1997) test for degree of support, assuming that degree of support is a probability function conforming to Kolmogorov's axioms.
v. Recently Kaplan (1996: 16-18) has produced a general proof that our personal degrees of confidence in the propositions we entertainought to conform to the constraints of a probability function, on the basis of some plausible assumptions about the structure of rational preferences, within the context of an oversimplified postulate about basic values. Kaplan's proof avoids some of the unrealistic assumptions of the more crude "Dutch book" argument first advanced by Ramsey (1990/1926) and found for example in Skyrms (1967). Savage (1972 [1954]) produced a more general proof than Kaplan's, one which does not involve any constraints on what an agent values. As Kaplan (1996) points out, the probability calculus imposes only weak constraints on our assignment of degrees of confidence to propositions; a comprehensive epistemology would impose additional constraints.
vi. The complications in developing this idea are that an agent's value system does not simply equate value with money and that an agent may have an aversion to, or a liking for, gambling which would distort the effect of their degree of confidence in a proposition.
vii. One may appreciate this fact more readily if one expands the prior likelihood

p(E/K) in Bayes’ theorem, using the probability calculus, to get: [p(E/H & K) x p(H/K)] + [p(E/~H & K) x (1 – p(H/K))]. Suppose the evidence E is highly likely if the hypothesis is true but highly unlikely if the hypothesis is false. For example, let p(E/H & K) = 1 and p(E/~H & K) = .0008. Now suppose that the prior probability of the hypothesis is very low; for example, let p(H/K) = .0001. Then p(H/E & K) = p(H/K) x p(E/H & K) x {[p(E/H & K) x p(H/K)] + [p(E/~H & K) x (1 – p(H/K))]\} = .0001 x 1 x {[1 x .0001] + [.0008 x (1 -.0001)] = .0001 x {.0001 + [.0008 x .9999]\} = .0001 \'f7 \{.0001 + .0008\} = .0001 x .0009 = 1/9 = .1111. So, even though the evidence is bound to occur if the hypothesis is true and highly unlikely if the hypothesis is false, the posterior probability of the hypothesis, given the evidence, is only .1111.

viii. For their comments on earlier drafts of this paper, I would like to thank Howard Simmons, Roderic Girle, Francisca Snoeck Henkemans, Sally Jackson, Robert H. Ennis, two anonymous referees for the Canadian Philosophical Association, and above all Robert C. Pinto, who produced a challenging commentary when I presented an earlier version of this paper at the University of Windsor, and who saved me from some embarrassing technical errors. The aforementioned discussants are of course not responsible for any flaws that remain.

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