ISSA Proceedings 2002 – Linked And Independent Premises: A New Analysis

Most introductory logic and critical thinking textbooks include a discussion of linked and independent premises. The core intuition underlying this distinction is clear. In some arguments, the premises work together as a logical unit in such a way that the amount of support offered by one or more of the premises is dependent on the other(s). Example:

Case 1

1. All members of the Oakwood Society are over 50 years old.
2. Bert is a member of the Oakwood Society.
3. Therefore, Bert is over 50 years old.

Here, neither of the premises provides any support for the conclusion without the other. Taken together, however, the premises validly imply the conclusion. Thus, the premises interact to produce a degree of support that is not simply the sum of the supports of the individual premises. Premises of this sort are said to be *linked***[i]**.

In other arguments, the premises work completely separately and independently of one another, in such a way that the degree of support they provide for the conclusion remains the same even if some or all of the other premises are omitted or assumed to be false. Example:

Case 2

1. Harry's car has a flat tire.
2. Harry's right leg is in a cast.
3. Harry's driver's license was recently suspended.
4. Therefore, Harry won't drive his car to the game.

In this argument, each of the premises would continue to provide the same amount of support for the conclusion even if the other premises were omitted or assumed to be false. To be sure, the premises do "work together" in a sense: the

overall strength of the argument would be reduced if one of the premises were suspended or knocked down. However, the premises "work together" only in a purely evidence-accumulating way. Unlike linked premises, they don't interact to form a single logical unit that provides a degree of support that is generally much greater than the combined support of the premises considered separately. Premises of this sort are said to be *independent***[ii]**.

Although the basic intuition underlying the linked/independent distinction is clear, there is considerable disagreement in both the textbooks and in the scholarly literature about how exactly these notions should be defined. In this paper I shall argue that all the leading textbook and scholarly accounts of the distinction are flawed, and I shall propose an alternative way of formulating the distinction.

1. Textbook Accounts

There are three leading accounts of the linked/independent distinction offered in logic and critical thinking textbooks. These are:

– the falsity/no support test
– the omission/no support test
– the omission/diminished support test

Let's examine each of these in turn.

A. The Falsity/No Support Test

Probably the most popular test is the falsity/no support test, offered among others by Copi and Cohen (1998: 45), Moore and Parker (2001: G-3-4), Reichenbach (2001: 165), Bickenbach and Davies (1997: 71), and Kelley (1988: 87).

Kelley offers perhaps the clearest statement of the test. He writes:

"In order to tell whether a set of premises is additive [linked] or not, we look at each premise separately, and ask whether it would support the conclusion by itself, without the other premises. The other side of the coin is to ask what would happen if one of the premises were false. Would that destroy the whole argument? Then the premises are additive; they depend on each other to support the conclusion. Or would part of the argument remain standing? Then the premises are nonadditive [independent]; each supports the conclusion independently" (Kelley 1988: 87).

Stated more precisely:

The falsity/no support test: A set of premises is *linked* just in case if any one of the premises were false, none of the other premises would provide any support for the conclusion. A set of premises is *independent* just in case if any one of the premises were false, then at least one of the remaining premises would continue to provide at least some support for the conclusion.

One problem with this test is that it is doubtful whether it makes sense to suppose that certain premises are false. Consider:
Case 3
1. All bachelors are males.
2. Max is a bachelor.
3. So, Max is a male.

Since (1) is necessary truth, it isn't clear what sense it makes to "assume" that it is false, or what follows logically from such an "assumption." (Standardly, counterfactuals with necessarily false antecedents are regarded as vacuously true.) Perhaps for this reason, most textbook writers prefer to speak of *omitting* premises in argument diagramming rather than assuming that they are false.

The falsity/no support test also runs into more obvious kinds of problems. Consider this argument in which one of the premises is only partially dependent on the other:
Case 4
1. All Chinese have brown eyes.
2. Xu is Chinese.
3. So, probably Xu has brown eyes.

Suppose the first premise is false. Then, because the second premise continues to provide relevant support for the conclusion, the premises count as independent on the falsity/no support test. But, intuitively, the argument is linked.

Finally, consider arguments in which the premises are irrelevant to the conclusion:
Case 5
1. Grass is green.
2. The North won the American Civil War.
3. So, Bush is the U.S. president.

According to the falsity/no support test, this argument is linked, since neither

premise would provide any support for the conclusion if the other were false. Yet, intuitively, the premises are not objectively dependent on one another.

B. The Omission/No Support Test

Another leading textbook account of the linked-independent distinction is the omission/no support test, endorsed by Govier (2001: 51-52), Freeman (1988: 178), Johnson (1999: 13), Moore (1993: 40), and Rudinow and Barry (1999: 97), among others. The test can be stated as follows:

Omission/no support test: A set of premises is linked just in case if any one of the premises were omitted, none of the other premises would provide any support for the conclusion. A set of premises is independent just in case if any one of the premises were omitted, then at least one of the remaining premises would continue to provide at least some support for the conclusion.

This test avoids the problems that the falsity/no support test encounters by asking argument diagrammers to assume (arguendo) that certain premises are false. But it faces other objections, including the following. Consider:

Case 6

1. The first letter of X's first name is "T"
2. The second letter of X's first name is "e".
3. The third letter of X's first name is "d".
4. Therefore, X's first name is probably Ted.

Intuitively, this argument is linked, because the premises provide strong support when taken collectively but very little support when considered individually. According to the omission/no support test, however, the premises are independent, because if any one of the premises were omitted, each of the remaining premises would continue to provide at least some support for the conclusion.

The omission/no support test also runs into problems with arguments that include countervailing premises, i.e., premises that, considered individually, provide evidence contrary to an argument's conclusion, but which nevertheless play an integral role in the argument as a whole. Consider:

Case 7

1. On Monday, I interviewed 40 Wexford College students and 32 of them were Republicans.

2. On Tuesday, I interviewed 10 Wexford College students and 4 of them were Republicans.
3. Therefore, most Wexford College students are probably Republicans.

According to the omission/no support test, this argument is linked because if the first premise were omitted, the second premise wouldn't provide any support for the conclusion at all. On the other hand, the premises are also independent, because if the second premise were omitted, the first premise would continue to support the conclusion**[iii]**.

C. The Omission/Diminished Support Test
Another leading test is the omission/diminished support test, endorsed, among others, by Hurley (2000: 64-65) and Layman (1999: 73). The test can be stated as follows:
The omission/diminished support test: A set of premises is linked just in case if any of the premises were omitted, the support provided by the other(s) would be diminished or destroyed. A set of premises is independent just in case if any of the premises were omitted, the support provided by the other(s) would not be diminished or destroyed.
This test works well with standard sorts of arguments, but it fails with various kinds of nonstandard arguments, including arguments with irrelevant or redundant premises.

Consider this typical argument with irrelevant premises:
Case 9
1. All dogs are mammals.
2. Some mammals are insects.
3. So, some insects are dogs.

Intuitively, this argument, like all categorical syllogisms, is linked. Yet, since neither premise would lose any power to support the conclusion if the other were omitted, the premises are independent according to the omission/diminished support test.

Further, consider this argument with redundant (i.e., logically superfluous) premises:
Case 10
1. If Joe is an uncle or a father, then Joe is a male.

2. The person referred to in the first premise is an uncle.
3. The person referred to in the first premise is a male.
4. So, Joe is a male.

On the omitted/diminished support test, this argument is linked, since if (1) were omitted, premises (2) and (3) would no longer provide any support for the conclusion. But the argument is also independent, since if (2) or (3) were omitted (but not both), the argument would still provide logically conclusive support for the conclusion.

2. Scholarly Accounts

The deficiencies of the various textbook accounts of the linked/independent distinction have been widely noted in the scholarly literature, and various attempts have been made to state the distinction more adequately and precisely. Here, I shall examine two such attempts.

A. Yanal's Account

In various writings, Robert J. Yanal has defended an account of the distinction that turns on the notion of a set of premises "summing in the ordinary way" (Yanal 1988: 43, 53-55; 1991: 140). Consider, by way of explanation, the following example offered by Conway (1991: 150):

Case 11

1. Sharpshooter A will shoot at Herman, and she hits her target 80% of the time.
2. Sharpshooter B will shoot at Herman, and she hits her target 90% of the time.
3. Therefore, Herman will be shot.

How do we determine how strongly the premises, taken together, support the conclusion? Clearly, we cannot just add the two probabilities together, for that would mean that the premises provide more than 100% support for the conclusion, which is impossible. So how should the premises be totaled?

According to Yanal, we should proceed as follows: Take the degree of support provided by the first premise (0.8). Then multiply the degree of support provided by the second premise (0.9) by the "unknown" left over from the first premise (0.2); thus, 0.9 x 0.2 = 0.18. Finally, add the two numbers together (0.8 + 0.18 =0.98). This means that there is a 98% chance that Herman will be shot. Assuming that the premises are completely independent, this seems to be the correct result [**iv]**.

When the premises of an argument "total" in the way they do in Case 11, Yanal

says, they may be said to “sum in the ordinary way.” Given this clarification, we are now in a position to state Yanal’s proposed test.

Yanal’s Summing Test: Two or more premises are independent when each premise provides at least some support for the conclusion and the premises sum in the ordinary way. Two or more premises are linked when they do not sum in the ordinary way but, instead, work together to make the overall strength of the argument much greater than they would if they were considered separately.

While this test seems to capture something intuitively right about linked/independent distinction, it confronts many objections. It assumes, dubiously, that it is possible to assign specific probability values to individual premises. It also fails to apply to many kinds of bad arguments. Consider:

Case 12

1. No Archbishops are professional wrestlers.
2. No professional wrestlers are grand chess masters.
3. So, no Archbishops are grand chess masters.

Intuitively, this argument, like all categorical syllogisms, is linked. However the premises provide no relevant support for the conclusion. Thus, according to Yanal’s test, the premises are neither linked nor independent.

Yanal’s test also fails when the support provided by the premises is only slightly greater if the premises are interpreted as linked rather than independent. Consider:

Case 13

1. X is a 4-year-old, 3-foot-tall paraplegic.
2. No NBA player is shorter than 5″10.
3. Therefore, X is not an NBA player.

Intuitively, this argument is linked because the premises, taken together, validly imply the conclusion. However, the first premise would continue to provide extremely high support for the conclusion (at least 99.99%) even if the second premise were omitted. Thus, according to Yanal’s test, the argument is not linked, because the premises, treated as a logical unit, do not make the overall strength of the argument much greater than they would if they were considered separately. Nor is the argument independent on Yanal’s test, because the second premise, considered separately, provides no relevant support for the conclusion.

Finally, Yanal's test yields counterintuitive results with many arguments that include countervailing premises. Example:

Case 14

1. CJ wears a beard.
2. CJ can bench-press 400 lbs.
3. CJ smokes cigars.
4. CJ enjoys knitting.
5. On balance, CJ is probably a man.

Intuitively, the premises of this argument are independent. However, because only some of the premises support the conclusion and the argument does not sum in the ordinary way, the argument counts as neither independent nor linked on Yanal's test.

B. Walton's Degree of Support Test

While conceding that no test works in every case, Douglas Walton proposes what he calls the "degree of support test" as the best available account. He states the test as follows:

The Degree of Support Test: "First, block one premise out of your mind, and then ask what degree of support the other premise (if true) gives (by itself) to the conclusion. Then, reverse the process, and block the other premise out of your mind, asking what degree of support the first premise (if true) gives (by itself) to the conclusion. Then, you add these two weights of support together, and ask what degree of support both premises together give to the conclusion. If there is a significant jump from the first joint degree of support to the second, the argument is linked. Otherwise, it is convergent" (Walton 1996: 181-182).

This test is similar to Yanal's account except that it avoids Yanal's problematic claim that the crucial difference between linked and independent arguments is that linked arguments do, and independent arguments do not, sum in the ordinary way.

However, Walton's test is vulnerable to many of the same objections that undermined Yanal's account. Since there is no "significant jump" in the joint degree of support when arguments with irrelevant premises are interpreted as linked rather than independent, all such arguments are counted as independent. Yet, intuitively, as we saw, arguments with irrelevant premises like those in Case 12 are linked. Moreover, Walton's test, like Yanal's, yields counterintuitive results with arguments like those in Case 13, where the premises provide only slightly

increased support when interpreted as linked rather than independent, and with arguments like those in Case 14, which feature countervailing premises.

In short, none of the leading textbook or scholarly accounts of the linked/independent distinction appear to be successful. Some logicians, most notably David Conway (Conway 1991:156) have concluded that all attempts to draw a clear, workable distinction between linked and independent premises have failed, and urge that we drop the distinction altogether. (Presumably, Conway would prefer to treat all arguments as explicitly or implicitly linked, seeing all apparently independent arguments as enthymemes with one or more implied linking premises.) However, I shall argue that a reasonably clear, intuitively sound distinction can be drawn between the linked and independent premises.

3. A New Account

Intuitively, a premise, P, is linked to another premise, P*, when a dependency relationship exists between P and P*, that is, when P is dependent upon P* for its degree of support, or P* is dependent upon P for its degree of support, or (as is typically the case) the premises are interdependent, each depending on the other for its degree of support. Put otherwise, two premises are linked when one or both affects the level of argumentative support of the other; otherwise, they are independent. But how can we make this intuitive notion more precise?

Consider an analogy. Imagine two lights, A and B, that are linked in the sense that one or both of the lights is dependent on the other for its ability to shine. Clearly, there are a variety of ways in which this dependency relationship could be manifested. One possibility is that if A were eliminated, B wouldn't shine at all, and if B were eliminated, A wouldn't shine at all. Another is that one or both lights might be dimmed (but still able to shine) if the other were eliminated. Still a third possibility is that one or both lights would be brightened if the other were eliminated. In fact, with two lights and four possible dependency relationships (no shine, dimmer, brighter, and no change), there are fifteen possible ways in which a dependency relationship could exist between the lights. Only if neither light is dependent on the other for its ability to shine are the lights independent.

My suggestion is that the lights analogy closely models the possible ways in which two premises can be linked or independent in arguments. Two lights, A and B, are linked just in case A affects (i.e., destroys, weakens, or strengthens) the ability of B to shine, or B affects the ability of A to shine, or A and B affect one another. Two lights that aren't linked are independent of one another. Similarly, two

premises, P and P* are linked just in case P affects (i.e., destroys, weakens, or strengthens) the ability of P* to support the conclusion, P* affects the ability of P to support the conclusion, or P and P* affect one another. Two premises that aren't linked are independent of one another.
We are now in position to state our proposed test of the linked/independent distinction, which I shall call the Dependency Relation Test (DRT). I shall first state a preliminary version of the test and then a revised version.

DRT-1:
Two premises, P and P* are linked if and only if the omission of P would affect (i.e., increase, diminish, or destroy) the amount of support P* provides for the conclusion; or the omission of P* would affect the amount of support P provides for the conclusion; or both. Two premises, P and P*, are independent if and only if they aren't linked.

This account, I suggest, successfully handles most of the objections that proved problematic for the other accounts, including arguments with necessarily true, partially dependent, or countervailing premises. However, there are two kinds of arguments that raise problems for DRT, namely, arguments with irrelevant premises and arguments with redundant premises. Consider first:
Case 15
1. All cats are mammals.
2. No beetles are cats.
3. So, all beetles are mammals.

In this argument, since neither premise seems to affect the degree of support offered by the other (the premises provide no support for the conclusion regardless of how they are interpreted), my test implies that the premises are independent. Intuitively, however, they are linked.

Consider next:
Case 16
1. If either Fred is an uncle or Fred is a father, then Fred is a male.
2. Fred is an uncle.
3. Fred is a father.
4. So, Fred is a male.

Here, one of the premises is redundant. The argument would still be logically

valid if either (2) were omitted and (3) retained, or if (3) were omitted and (2) retained. DRT-1 thus implies that (2) and (3) are not linked, since the argumentative force of the remaining premises would not be affected if either were omitted. Yet suppose the arguer in Case 16 mistakenly believes and intends that the premises are linked, perhaps because he believes that adding "back-up" premises to an already valid argument can make the argument stronger. In that case, it is plausible to regard the argument as linked, much as we regard an argument as deductive if the arguer obviously intended the argument to be deductive, even if the conclusion plainly does not follow validly from the premises.

What examples like those in Cases 15 and 16 show is that any satisfactory account of the linked/independent must take into account the (actual or reasonably imputable) intentions of the arguer. Arguments like those in Cases 15 and 16 are rightly treated as linked, I suggest, not because there is any actual dependency relationship between the premises, but because the arguer presumably believed that there was such a relationship. In this respect, the linked/independent distinction is similar to the deductive/inductive distinction. Both ultimately turn on the messy and often only guessable issue of arguers' intentions.

Given the crucial role of arguer's intentions, my account must be revised as follows:
DRT-2:
The Dependency Relation Test: Two premises, P and P* are linked if and only if the arguer believes (1) that the omission of P would affect (i.e., increase, diminish, or destroy) the amount of support P* provides for the conclusion; or (2) that the omission of P* would affect the amount of support P provides for the conclusion; or both (1) and (2). Two premises, P and P*, are independent if and only if they aren't linked.

DRT-2 correctly implies that that the premises in Cases 15 and 16 are linked.

A. An Objection: Too Much Guesswork?
In closing, I would like to consider a likely objection to my proposed account, namely, that it is unsatisfactory because it involves too much guesswork.

Consider this example offered by David Conway (Conway 1991: 150):
Case 17

1. Harvey handles cobras barehanded and 80% of people who handle cobras barehanded die young.
2. Harvey drinks antifreeze for breakfast and 90% of people who drink antifreeze for breakfast die young.
3. Therefore, Harvey will die young.

Is this argument linked or independent? On the face of it, it looks virtually identical to the sharpshooter argument in Case 11, which we treated as independent. But appearances may be deceiving, as Conway points out. Suppose that, unknown to anyone, drinking antifreeze daily makes one partially immune to the effects of cobra venom. In that case, the premises logically interact with one another, and thus are linked. Conway's point is that standard accounts of the linked/independent distinction cannot be relied upon to give correct evaluations of premises that appear to be independent but aren't (Conway 1991: 150-151).

Conway directs his objection primarily against Yanal's summing test, but it can be applied to all of the tests we have examined. The falsity/no support test and the omission/no support test will give the wrong answer, mistakenly counting the argument as independent, whereas the omission/diminished support test, Yanal's summing test, and Walton's degree of support test can't be reliably applied, since, by hypothesis, is isn't known whether the premises are independent, although, by hypothesis, they are not.
Does Conway's objection also apply to my account? Not directly. For on my account, what determines whether an argument is linked or independent isn't whether the premises work as a logical unit but whether the arguer believes that they do. But of course this only shifts the locus of guesswork. In many cases, we have no real evidence what a particular arguer may have believed or intended, but must fall back on the hypothetical "typical" or "reasonable" arguer who figures so ubiquitously in informal logic. Thus, instead of guessing whether a set of premises like those in Case 17 interact in reality, we must guess whether a typical or reasonable arguer would believe that they do. And often, Conway might object, we will guess wrong.

Does the fact that my account doesn't always yield clear, determinate answers show that the account is flawed? Hardly, for any plausible account of the linked/independent distinction will sometimes involve a significant amount of guesswork. Consider:
Case 18

1. Bob is stubborn.
2. Bob is a Taurus.
3. Therefore, Bob won't make a good mediator.

Are the premises in this argument linked or independent? That depends on how the argument is interpreted. Are (1) and (2) offered as separate, freestanding reasons for (3)? In that case, the premises are independent. Is (1) offered as a reason for believing (2), or (2) offered as a reason for (1)? In that case the premises are either independent or neither linked nor independent, depending on whether single premises are counted as independent or as neither linked nor independent. Or is the argument in fact an enthymeme, with, say, Most stubborn persons are not good mediators operating as an implied premise? In that case, two of the premises are linked and the other is not. The point is that there are inherent unclarities in argument structure that present difficulties for all standard approaches to argument diagramming.

Of course, some approaches to argument diagramming in general, and the linked/independent distinction in particular, may produce more uncertainty than others. In particular, the falsity/no support and omission/no support tests generally produce more clear-cut results than the other tests we examined, in part because it is easier to determine when a premise provides no relevant support for a conclusion than it is to determine whether it provides stronger or weaker support than it does in conjunction with another premise. But the first virtue of a satisfactory account of the linked/independent distinction is that it be adequate, not that it be straightforward to apply. And as we have seen, the falsity/no support and omission/no support tests are far from adequate.

Granted, the Dependency Relation Test defended in this article is probably too complex to be taught in an introductory logic or critical thinking textbook. For pedagogical purposes, therefore, it might be better to present introductory students with a simpler account, such as the omission/diminished support test, which in my view is the least misleading of the standard textbook accounts. However, it should be clearly stated that a simplified account is being presented and that an adequate account is more complex**[v]**.

NOTES

[i] Other terms for "linked" include "conjoint," "dependent," "interdependent," and "additive".

[ii] Other terms for "independent" include "convergent" and "nonadditive."

[iii] I suspect that many of the confusions that bedevil the linked/independent distinction arise from confusions about what it means to "omit" a premise from an argument. Consider:

1. I promised Ann I would play tennis with her today.
2. Promises should always be kept, no matter what.
3. So, I should play tennis with Ann today, even though I have a splitting headache.

Suppose we omit the second premise in order to determine whether the premises are linked or independent. Intuitively, the first premise continues to provide some support for the conclusion. But no statement is probable with respect to another except in conjunction with a stock of relevant background information. So presumably the relevant antecedent is not:

(A-1) If any of the premises are omitted and absolutely nothing else is stated or assumed that would make the remaining premise(s) relevant to the conclusion

For if this is what it means to "omit" a premise, then all two-premise intuitively independent arguments would turn out to be linked.

But if this isn't the right way of formulating the relevant antecedent, what is? Perhaps this:

(A-2) If any of the premises are omitted and nothing similar is stated or assumed in its place that would make the remaining premise(s) relevant to the conclusion

I suspect something like (A-2) is what advocates of the various omission tests of the distinction have in mind. If so, there are obvious difficulties to overcome in spelling out what counts as a "similar" premise. Amazingly, this crucial issue appears not to have been discussed in the relevant literature.

[iv] Think of it this way: If sharpshooter A takes 100 shots at Herman, she will hit Herman 80 out of 100 times. This leaves 20 times when A's bullets will miss Herman. But, since sharpshooter B is 90% accurate, in 18 out of those 20 cases, B's bullets will hit Herman. Thus, in only two cases out of every 100 will both bullets miss Herman.

[v] My thanks to Robert J. Yanal and Bill Drumin for helpful comments on earlier versions of this paper.

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