ISSA Proceedings 2010 – Epistemic Foundationalism and Aristotle’s Principle of the Absolute

1. Turtles all the way down
According to an ancient Hindu myth, the earth is a flat disc resting on the back of a tiger. The tiger stands on an elephant, and the elephant in turn stands on the carapace of Chukwa, a gigantic tortoise. The obvious question ‘What is Chukwa standing on?’ was already posed by John Locke in the seventeenth century and again by William James two centuries later (Locke 1959, p. 230, p. 392; James 1931). Not that Locke and James were particularly interested in an answer: it seems that they simply wanted to make fun of a cosmogony that reduced the world to an exotic version of the Grimm’s Bremen Town Musicians.

A variant of this myth is to be found in the first lines of Stephen Hawking’s bestseller *A Brief History of Time:*
“A well-known scientist (some say it was Bertrand Russell) once gave a public lecture on astronomy. He described how the earth orbits around the sun and how the sun, in turn, orbits around the centre of a vast collection of stars called our galaxy. At the end of the lecture, a little old lady at the back of the room got up and said: ‘What you have told us is rubbish. The world is really a flat plate supported on the back of a giant tortoise.’ The scientist gave a superior smile before replying: ‘What is the tortoise standing on?’ ‘You’re very clever, young man, very clever’, said the old lady. ‘But it’s turtles all the way down!’ (Hawking 1988, p. 1)

The idea of an infinite sequence of turtles supporting the earth is, if anything, even more absurd than that of one reptile doing the job. An infinite set of turtles, assuming that they could exist, would after all still need to stand on some ground. What could that ground be? Not a turtle, for every turtle has another turtle under its feet. But it can be nothing other than a turtle, after all *it’s turtles all the way* *down.*

Throughout the ages philosophers have thought it obvious that such unending series of reasons are absurd. Whether it be turtles that stand on other turtles, or events that arise from other events, or actions performed for the sake of other actions: in all cases it is thought to be incoherent that such a series might consist of an infinite number of steps. At some point the series will have to stop: either at a turtle that supports but is not supported, or at an event that causes but is itself uncaused, or at an action for the sake of which all other actions are performed, but that itself is performed for its own sake.

A veto on a *regressus in infinitum* is a *Leitmotiv* that resounds down the ages throughout the annals of philosophy. Innumerable proofs of God's existence depend on this proscription. Even philosophers who expressed doubts about it, such as Kant in his discussion of the antinomies, thought it better to go quietly along with the embargo. Clearly the tendency to call a halt to threatening endlessness is deeply anchored in our cognitive apparatus.

In his inaugural dissertation as an extraordinary professor in Amsterdam, the logician and philosopher Evert Willem Beth subjected this tendency to searching scrutiny (Beth 1946). The prohibition on infinite series or sequences is, according to Beth, a crucial component of traditional metaphysics. Moreover, Beth sees much evidence that this ban on infinite sequences is mostly implicit and not openly addressed. He himself goes on to make it fully explicit in what he calls 'Aristotle's Principle of the Absolute' (APA):
"Suppose we have entities *u* and *v*, and let *u* have to *v* the relation *F*; then there is an entity *f*, which has the following property: for any entity *x* which is distinct from *f*, we have (i) *x* has the relation *F* to *f*, and (ii) *f* has not the relation *F* to *x*." (Beth 1968, p. 9).

In symbols:
(\$*u*) (\$*v*) *F*(*u*, *v*) → (\$*f*) ("*x*) [*x* ≠ *f* → { *F*(*x*, *f*) & ¬*F*(*f*, *x*)}].

Applied to our turtles, this principle would say that, if turtle *a* is supported by turtle *b,* and *b* by *c,* and so on, there must be a turtle *f* that (perhaps indirectly) provides support for turtles *a,b,c,* and so on, but which itself is not supported by any of the other turtles.

Of course no-one takes this turtle example seriously. And indeed, the illustrations of the principle that Beth gives of APA are historically more responsible. Here are three of them.

(1) Interpret '*a* has the relation *F* to *b*' as '*a* comes into being through *b*'. Then the absolute entity *f* is that through which all others come into existence, but which does not exist by virtue of any of the other entities. Beth argues that this *f* has all the characteristics of the *archè* in the sense of Presocratic philosophy.
(2) Interpret '*a* has the relation *F* to *b*' as '*a* is desired because of *b*'. Now *f* becomes the *summum bonum,* i.e. the Supreme Good in the sense of Aristotle and of the Mediaeval church fathers.
(3) Interpret '*a* has the relation *F* to *b'* as '*a* is moved by *b*'. In this case *f* is Aristotle's Unmoved Mover, i.e. that which sets all else in motion but remains itself unmoved.

However often APA has been applied in philosophy, including natural philosophy (for Beth suggests that Isaac Newton uses it in his arguments for absolute space), the principle itself is of course invalid. This can be simply shown by giving a counterexample. Take for *a,b,c,…* the integers (positive, negative, and zero), and interpret '*a* has the relation *F* to *b*' as ' *a* is larger than *b*'. Then APA leads to the false conclusion that there is an integer, *f*, that is smaller than any of the other integers.

The fact that APA is not valid does of course not imply that it sometimes cannot lead to true statements. Indeed it can. If one changes the domain of *a,b,c,…* from that of the integers to that of the natural numbers, 1,2,3,… only, then there is indeed a natural number that is not larger than any of the others, namely 1.
Accordingly, the lesson that Beth draws is not that APA is worthless, but that it has to be applied with care. It depends on the nature of the relation *F*, and nature of the domain *a,b,c,…* whether APA leads to a correct conclusion or not. Under the interpretation of *F* as 'is larger than' the conclusion is correct if *a* and *b* are the natural numbers, but incorrect if *a* and *b* are the integers.

2. A chain of reasons
Why am I explaining this matter? Not merely to laud Evert Willem Beth, but to draw attention to recent developments in epistemology.
An important question in modern epistemology is what it means to say that a given belief or proposition is *justified* by another belief or proposition. In an epistemic chain a proposition p_0 is justified by another proposition p_1 that, in its

turn, is justified by a third proposition p_2, and so on. Another way of expressing this is by saying that a reason for p_0 is p_1, and a reason for p_1 is p_2, and so on. Exactly as in the case of the turtles and the examples of Beth, the question arises whether this chain can extend indefinitely. Does an infinite sequence of reasons make sense?

Most epistemologists assume without much debate that such an endless chain is absurd. The majority of these philosophers insist that the chain must terminate in a ground that is not justified by another proposition, but is true, or is probably true, *tout court.* These are the epistemic *foundationalists,* who number among their ranks giants like Plato, Aristotle, Descartes, Hume, Berkeley, and in the first half of the 20th century C.I. Lewis and Moritz Schlick. In the second half of the 20th century foundationalism lost some ground, but it has in the last few decades made a strong comeback, witness books with titles such as *Resurrecting Old-Fashioned Foundationalism* (DePaul 2001).

This comeback should not surprise us. For foundationalism is a position that has a great intuitive appeal. Indeed, what is more natural and obvious than the idea that our knowledge is grounded, that one cannot go on and on with justification. Moreover, it seems that, as Jonathan Dancy has it: "if all justification is conditional …, then nothing can be shown to be actually … justified" (1985, p. 55). A proposition p_0 whose justification is conditional on that of p_1, whose justification is conditional on that of p_2, and so on, *ad infinitum*, is not justified at all, so it seems. If we want to justify p_0 at all, the sequence of justificatory propositions will have to terminate at a source from which the ultimate justification springs (cf. Gillet 2003, p. 713).

However, choosing foundationalism means no more nor less than opting for Aristotle's Principle of the Absolute in the field of epistemic justification. And the lesson of Evert Willem Beth was that this principle, however intuitively plausible it may be, does not always lead to a correct conclusion. Sometimes it does, but sometimes it does not: it all depends on the domain in which it is applied, and the relation between the elements in the domain. Let us look more closely both at the domain and the relation. We will then find out whether APA applies or not.

3. Truth and probability

The nature of the domain that applies to an epistemic chain is obvious enough: it is that of propositions or beliefs in propositions. At first sight the identity of the relation seems clear too, it is that of epistemic justification. In an epistemic chain, proposition p_n is justified by proposition p_{n+1}, so p_{n+1} is an epistemic reason for p_n.

The matter is however not so simple. What exactly do we mean when we say that one proposition is a reason for another? What is the precise nature of the justification relation? Today the answer to this question differs from what used to be thought. Epistemologists in the past generally supposed that justification is some sort of inference: to say that p_n is justified by proposition p_{n+1} meant for the traditional epistemologist that the truth of p_n is inferred from the assumed truth of p_{n+1}. Modern epistemologists have a different approach. They stress “the widely accepted point”, in the words of Jeremy Fantl, “that justification comes in degrees” (Fantl 2003, p. 537). In other words, justification is seen as a gradual concept: it can be more or less. Consequently, present-day epistemologists are more sympathetic to the view that justification is to be understood in probabilistic terms rather than as a form of inference. Below we shall consider chains of justification both according to the old interpretation of justification, and according to the new probabilistic interpretation. We will see how epistemic chains of infinite length fare in both cases.

First the old understanding: epistemic justification as a form of inference. The inference may be deductive or inductive, but here we will concentrate on deductive relations.**[i]** Is it coherent to maintain that a chain of deductive implications could go on and on indefinitely? We saw that Dancy thinks not. If one justifies p_0 by pointing out that it follows deductively from p_1, and p_1 by showing that it deductively follows from p_2, and so on, then that means, according to Dancy, that there is no justification of p_0 at all.

Dancy presumably means that we *can never know* if p_0 is true or false if the chain of implication is infinite in extent. If this is what he means, he is not necessarily right. It all depends on what the negations of the various propositions in the chain imply. If p_{n+1} implies p_n and Øp_{n+1} implies Øp_n for all n=0,1,2,…, then the chain is one of bi-implications, and so all the propositions are together either true or false. Indeed, in this case we would not know which was the case. But if Øp_{n+1} implies p_n instead of Øp_n for all n=0,1,2,…, then all the propositions in the chain are true. This may not be a very interesting situation, for all the propositions would be tautologies, but it is a case in which we would know the truth value of p_0.

What happens to an infinite chain when justification is interpreted as a probabilistic relation that satisfies the Kolmogorov axioms, as many modern epistemologists are wont to do? The great majority of contemporary epistemologists still think that an infinite chain of justification makes no sense, but not everyone agrees as to why this is so. Sometimes it is thought that the probability associated with a proposition is necessarily *undefined* if the chain is infinitely long (Dancy), and sometimes it is claimed that it is defined, but is necessarily *zero* (Lewis 1929, pp. 327-328; Lewis 1952, p. 172). In the first case the probability of p_0, say $P(p_0)$, has no value, and in the second, $P(p_0) = 0$.**[ii]**

In recent years I have argued that these claims are incorrect (Atkinson & Peijnenburg 2006; 2009; Peijnenburg 2007; Peijnenburg & Atkinson 2008). Modern foundationalists of all stripes, whether they think that an infinite series of probabilistic relations must lead to probability zero or to none at all, are mistaken. Not only is it possible that such an infinite series leads to a definite and sensible value, it is in fact a very common situation. The assumption that we need to make is that the conditional probabilities along the chain obey the following inequality:

$P(p_n|p_{n+1}) > P(p_n|\O p_{n+1})$,

for all n. This is a very natural assumption indeed. It states that p_n is more likely to be true if p_{n+1} is true than if p_{n+1} is false, and thus that p_{n+1} makes probable p_n. For p_0 to be *justfied* we require in addition that the resulting probability $P(p_0)$ is greater than $P(\O p_0)$, and often one requires more than this, namely that $P(p_0)$ be greater than some agreed upon theshold of acceptance, say 0.9.

Under the above inequality, the usual situation is that $P(p_0)$, and indeed all the unconditional probabilities $P(p_n)$ in the chain, have well-defined, nonzero values. Aristotle's Principle of the Absolute thus generally fails in the case of chains of probabilistic justification. True, there *are* sequences of conditional probabilities in which $P(p_0)$ is undefined by the infinite series, and others where $P(p_0)$ is defined by the infinite series but is zero. But far from being always the case, such sequences are demonstrably very exceptional special cases. The generic situation is that in which the unconditional probability of p_0 is well-defined and nonzero, even if the justification of p_0 consists of an infinite chain of conditional probability statements.

It will be clear that epistemic justification in a probabilistic context is much more interesting than it is when justification is conceived as implication. As we saw, in the latter case the possibility of an infinite series leading to a well-defined probability was restricted to an exceptional and not very interesting state of affairs. When the series is one of probabilistic justification, however, the matter is precisely reversed. Now it is the norm that an infinite series leads to a well-defined and significant probability, and exceptions are rare and not very important.

The reason that we can often complete an infinite probabilistic series is that the contribution from the conditional probabilities, $P(p_n|p_{n+1})$ and $P(p_n|Øp_{n+1})$, becomes smaller and smaller as n becomes larger and larger. This does not mean that the conditional probabilities themselves need to tend to zero, for they could even tend to one, or they may indeed become smaller: that is of no import. The essential thing is that their contribution to $P(p_0)$ becomes smaller and smaller as n increases, and that the infinite series of probabilities is always convergent. Elsewhere we have proved that the sum of the series indeed always converges and that it differs from $P(p_0)$, the probability of the target proposition, only in very exceptional cases (Atkinson & Peijnenburg 2010).

Another interesting consequence of these results is as follows. Suppose that the epistemic chain in a particular case *is* finite, but *very* long. Because of the finitude, there must be a last proposition, say p_{1000}, separated from p_0 by a 999 links. For the foundationalist p_{1000} is the ultimate ground on which the justification of p_0 rests. After all, it is p_{1000} that justifies p_{999}, and p_{999} that justifies p_{998}, and so on. To determine $P(p_0)$ we need to know not only all the conditional probabilities, but also the unconditional probability $P(p_{1000})$. At first sight this looks like grist to the foundationalist's mill, but the opposite is in fact the case! The numerical contribution of the probability $P(p_{1000})$ to $P(p_0)$ will generally be very tiny, for $P(p_{1000})$ is multiplied by a coefficient that involves all the conditional probabilities along the entire chain, and this coefficient is small. The lion's share of the contribution is provided by the conditional probabilities alone, without hardly any help from $P(p_{1000})$. The 'ground' p_{1000} may be very probable, even certain, $P(p_{1000}) = 1$, or very improbable, even absent, $P(p_{1000}) = 0$; all this makes very little difference to the calculated value of $P(p_0)$. It should be clear that this fact flies in the face of the foundationalist who insists that the series, and the probable truth of the proposition in question, is completely supported by one solid foundation.**[iii]**

In conclusion, I have claimed that an infinite chain of propositions that support one another epistemically is not absurd. The situation is however radically different if epistemic support is construed as implication on the one hand, or in probabilistic terms on the other. As we have seen, an infinite epistemic chain almost never leads to a *truth value* as such for a target proposition, but almost always to a *probability value.*

4. Two objections

One might cavil at the above conclusions in two ways. The first complaint could be to claim that I have merely shown there to be a conceptual, but not a physical possibility of an infinite epistemic chain. Is this not simply a mathematical trick? The second, related objection is that the argument tells us nothing about the world.

Concerning the first objection, it is of course true that we are not able to give unlimited reasons for our reasons *sub specie aeternitatis*. We are mere mortals who have only a limited time at our disposal. Unending epistemic chains in *this* physical sense are for practical reasons impossible. The interesting objections against infinite regression do not have to do with this physical impracticability, but rather an imagined conceptual impossibility (Post 1980). Surprisingly this does not refer to our inability to argue for or retain an infinite *number* of thoughts or reasons, for many foundationalists are quite happy to admit that this is in some sense possible. Fumerton, for example, admits roundly that "we do have an infinite number of beliefs" (Fumerton 2001, p. 7). What foundationalists deny is that all these beliefs could be tacked on to one another in an infinite chain in such a way as to lead to a well-defined (generally gradual, i.e. probabilistic) belief. They deny, in other words, that we can *complete* an infinite epistemic chain: "we cannot complete an infinitely long chain of reasoning" (Fumerton 2004, p. 150; 2006, p. 40). Or in the formulation of Robert Audi:
"For even if I could have an infinite number of beliefs, how could I ever know anything if knowledge required an infinite epistemic chain?" (Audi 1998, p. 183).

Above I asserted (and elsewhere I have demonstrated) that one can indeed have knowledge that presupposes an infinite epistemic chain; knowledge of a unique value for a probability, $P(p_0)$, is obtained from the consideration of an infinite, convergent series of conditional probabilities. Although *coming into possession* of the knowledge involved a conceptual exercise (namely the summation of a convergent series), the knowledge itself is not a mere conceptual business. It tells us something about the material world.

This brings us to the second objection. *Have* we really learned something about the empirical world if we have computed a probability on the basis of an infinite series? I can most readily explain how this can be so by giving an example. Imagine colonies of a bacterium growing in a stable chemical environment known to be favourable to a particular mutation of practical interest. The bacteria reproduce asexually, so that only one parent, the 'mother', produces 'daughters'. The probability that a mutated daughter descends from a normal, not mutated mother is known to be very small (say 0.02); but the probability that a mutated daughter descends from a mutated mother is on the other hand high (say 0.99).

Let p_n be the proposition: 'the ancestor in generation n, reckoned backwards from the present, was a mutant'. We are told further that each batch develops from a single, mutant ancestor. In this situation, in which the conditional probabilities are the same from generation to generation, $P(p_0)$ is equal to a geometric series that can be summed explicitly. Imagine a batch to be sampled after, shall we say, 150 generations since the seeding of the batch. The original great-great-grandmother, in generation 150 before the generation sampled, is known to be a mutant, so $P(p_{150}) = 1$, and we find that $P(p_0)$ is perfectly well defined: it works out to be 0.670.

Now we can just as easily calculate $P(p_0)$ on the assumption that the number of the preceding generations of bacteria was not 150, but infinite. We now have a geometric series with an infinite number of terms; but it can nevertheless be completed in the sense that its sum can be calculated exactly. We compute two thirds, which is only half a percent less than the 0.67 that we obtained using the assumption that the ancestor in the 150th generation was a mutant. Evidently we have made a very useful statement about the empirical world.

At this point, a foundationalist objecting to infinite chains might argue that our story about the bacterial colonies is not an example of infinitism at all. For no bacterium has an infinite number of ancestor bacteria, if only because of the fact of evolution from more primitive algal slime, which had evolved from earlier life forms, which sprang from inanimate matter, which originated in a supernova explosion, and so on, back to …. to what? To the Big Bang? But it seems that the Big Bang may well not represent a beginning, in view of the deformation of spacetime. The whole point here is precisely the question whether or not there was a starting point. The foundationalist's postulate that in the bacterial case there was a start begs the question.

NOTES

[i] Whereas deductive relations are clearly nonprobabilistic, inductive connections may be regarded as first steps towards a full understanding of justification in probabilistic terms. The latter remains however a matter of debate, since contemporary epistemologists are not in agreement on the sort of probability central to inductive justification.

[ii] As is well known, the concept of probability that satisfies the Kolmogorov axioms is open to several interpretations. For the purpose of this article it does not matter which interpretation is favoured, although it would be natural to think of probability as degree of belief.

[iii] John Turri has argued that foundationalists are not committed to finite epistemic chains, let alone to the idea that such chains must have a solid foundation (Turri 2009). Elsewhere it has been argued that Turri's argument rests on a confusion between the limit of a series and its ground (Peijnenburg and Atkinson, forthcoming).

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